THE ALPHABET RACE

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Visualize the 26 letters of the alphabet lined up like horses at the starting gate of an immensely-long racetrack. One minute after the start, letters O, N and E each advance one mile but the remaining 23 have not budged. During the second minute, T, W and O each advance one mile, placing O in the lead at two miles and W, E and T tied for second at one mile each. During the third minute, E advances two miles, while H, T and R advance one mile, placing E in the lead at three miles.

Since there are one thousand vigintillion (minus one) number names, this promises to be a long race indeed. Which letter will be the eventual winner? When does the lead change as the race progresses? The table at the end shows the development of the race for the first 100 minutes, focusing on the four leading letters T, I, N and E

O is a fast starter but fades rapidly; after 39 minutes it has gone only 39 miles. Although E holds the lead at 100 minutes, T actually led the race more than half the time, having taken the lead at the end of 39 minutes and relinquished it at the end of 85 minutes. D, a new letter joins the race at 100 minutes. A steady performer, it advances two miles every minute, but this is not enough to overtake the leaders; E, for example, adds one mile per minute to its previous average speed of 1.48 miles per minute.

Will E ever be overtaken? One might bet on I when the millions are reached, but this is not enough to matter, as shown by Frank Rubin in the Nov 1981 Colloquy. MILLION through QUINTILLION have no E's and two or more I's, enough to let I slowly gain, but SEXTILLION adds an E enabling it to pull away. Dan Hoey has written a computer program to find that E leads until the 1,908,414,049,538,005,261th minute when N passes it for the first time. E and N exchange the lead a total of 6 times in the quintillions, but E regains the lead in the sextillions, not to be bettered until the nonillions when 27 more exchanges take place. However, E has the last word, regaining the lead for good in the decillions and holding it the rest of the way.

Beyond vigintillion, one must create a number nomenclature in order to continue. In the *Book of Numbers* (Springer-Verlag, 1996), Conway and Guy have done just that. At 10^{63} there are three I-E exchanges, ending with I in the lead until 10^{240}, after which a number of I-N exchanges occur. But is anyone still interested?

	T	I	N	E		T	I	N	E
1			1	1	51	74	32	38	67
2	1		1	1	52	76	33	38	67
3	2		1	3	53	78	34	38	69
4	2		1	3	54	79	35	38	69
5	2	1	1	4	55	80	37	38	70
6	2	2	1	4	56	81	39	38	70
7	2	2	2	6	57	82	40	39	72
8	3	3	2	7	58	83	42	39	73
9	3	4	4	8	59	85	44	41	74
10	4	4	5	9	60	86	45	41	74
11	4	4	6	12	61	87	46	42	75
12	5	4	6	14	62	89	47	42	75
13	7	5	7	16	63	91	48	42	77
14	8	5	8	18	64	92	49	42	77
15	9	6	9	20	65	93	51	42	78
16	10	7	10	22	66	94	53	42	78
17	11	7	12	26	67	95	54	43	80
18	12	8	13	29	68	97	56	43	81
19	13	9	15	32	69	98	58	45	82
20	15	9	16	33	70	99	58	46	84
21	17	9	18	35	71	100	58	48	87
22	20	9	19	36	72	102	58	49	89
23	23	9	21	40	73	104	58	50	93
24	25	9	21	40	74	105	58	51	95
25	27	10	22	42	75	106	59	52	98
26	29	11	23	43	76	107	60	53	100
27	31	11	25	46	77	108	60	55	104
28	34	12	26	48	78	110	61	56	107
29	36	13	29	50	79	111	62	59	110
30	38	14	29	50	80	112	63	59	111
31	40	15	30	51	81	113	64	60	113
32	43	16	30	51	82	115	65	60	114
33	46	17	30	53	83	117	66	60	117
34	48	18	30	53	84	118	67	60	118
35	50	20	30	54	85	119	69	60	120
36	52	22	30	54	86	120	71	60	121
37	54	23	31	56	87	121	72	61	124
38	57	25	31	57	88	123	74	61	126
39	59	27	33	58	89	124	76	63	128
40	60	27	33	58	90	125	77	65	129
41	61	27	34	59	91	126	78	68	131
42	63	27	34	59	92	128	79	70	132
43	65	27	34	61	93	130	80	72	135
44	66	27	34	61	94	131	81	74	136
45	67	27	34	62	95	132	83	76	138
46	68	28	34	62	96	133	85	78	139
47	69	28	35	64	97	134	86	81	142
48	71	29	35	65	98	136	88	83	144
49	72	30	37	66	99	137	90	87	146
50	73	31	37	66	100	137	90	89	148