MAGIC SQUARE MAGIC

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I had heard the rumors for years. Somewhere, deep in the back woods, there existed an enclave of mentalists with some clever new prediction tricks. In fact, I finally learned that the group, the mysterious MOUSTERIAN cult, is led by my good friend Edith, known to readers of Word Ways as Scrubwoman Edith. Her last appearance on these pages was in the August 1999 issue in "Edith Plays Word Treblecross".

Mousterian headquarters is in the remote unincorporated village of St. Wordbuch, Maine. There Edith performs as the magicienne Madame Edith Camus-Brown. She has given me permission to describe several of the Mousterian tricks but she and I reserve all rights for their commercial reproduction.

Before the magic tricks are revealed, we must introduce Edith’s concept of a magic word square. The diagrams below and on the next page show examples of 3x3, 4x4 and 5x5 squares. Notice that each cell consists of a bona fide dictionary entry from the American Heritage Dictionary (3rd edition) of Merriam-Webster’s New International (2nd or 3rd editions). No letter is duplicated in any row or column; for the 4x4 and 5x5 squares, the main diagonals also have this property. These are truly “word” squares, not merely square arrangements of letters that happen to form words. They also each have their own magic constant—every row and column can be transposed into the name of the square.
Each of the squares has a mathematical counterpart. For ADONIS, set A=0, N=0, O=1, I=2, D=3 and S=6, then add the numerical values of each of the letter pairs. One obtains a square that is magic on the integers 0 through 8. The famous lo-shu square, reported by W.W.R. Ball and H.S.M. Coxeter in their book *Mathematical Recreations and Essays* to be many centuries old, can be had by adding one to each entry. Similarly, ASTEROID becomes numerically magic when the following letter values are added: A=0, S=0, E=1, I=2, O=3, T=4, R=8 and D=12. For MOUSTERIAN, set A=0, M=0, E=1, I=2, O=3, U=4, N=5, R=10, S=15 and T=20. In each case, the numerical magic constant is the number obtained by adding all letter values in the name of the square.

The same procedure can be used for magic cubes. In MOUSETRAP, every set of three words, in any of the three dimensions, transposes into MOUSETRAP. The cube can be turned into a numerical one by determining the misgraph of the configuration. The misgraph is the graph of letters that are never used together to form any of the words in the cells. Here the misgraph is the disjoint set OAU (Hawaiian, to mew as a cat), MRS and PET. Set O=0, M=0, P=0, A=1, U=2, R=3, S=6, E=9 and T=18 and we have a number magic cube on summing the letters.

It is worth mentioning that ASTEROID has many other cells that transpose into the magic constant. Any four cells symmetrically placed about the center point always yield ASTEROID—for example the four corners AS, TO, RE, and ID, or the four DO, IT, RA and ES, and so on. This square seems as fecund as the famous square etched in 1514 by Albrecht Durer in his work *Melancholia*.

ASTEROID also answers an old problem of Ozanam (1723) on how to arrange the 16 honor cards from an ordinary deck so that each row and column contain exactly one suit and one honor. Simply take the vowels as the suits and the consonants as the honors.
The mentalism that Edith performs can best be introduced using ASTEROID as a base. Obtain 16 blank cards and print the 16 words of ASTEROID on them. Edith has someone (let us suppose her friend Mark does the honors) arrange the cards in a 4x4 grid so that no letter is repeated in any row or column (ignore the diagonals). Mark does this with Edith’s back turned and when it is completed, he turns the cards over. Mark then is instructed to choose four cards, either four cards on the corners of some quadrilateral in the grid or four cards so that there is exactly one choice per row and one choice per column. The former is called a quad, and the latter is called a stagger. His original placement of the cards is called a mix. It is also possible to obtain a mix by interchanging any two rows or any two columns as often as one likes. Because of certain properties ultimately connected with the theory of determinants, under any mix the cards remain simpatico for Edith’s purposes.

Suppose Mark has selected a quad. He is then to choose one of the quad by marking it with a coin, and turns over the remaining three key cards of the quad. Edith is told the keys and can immediately name the marked card. Suppose Mark gives her the key cards ET-SO-AT of the quad. Edith has determined that in ASTEROID each letter in each half of the misgraph must occur exactly once with all others, or exactly four times, or exactly twice with some one other letters also occurring twice. Thus, for the vowels E-O-A-? the missing ? can only be I, and for the consonants T-T-S-? the missing ? can only be S, making the unknown card IS. The same technique works for a scatter. For example, suppose the keys DO-AT-ES are given. The vowel must be I and the consonant R, and so the marked word is IR.

![ASTEROID MISGRAPH](image)

It would not take long for this trick’s secret to be discovered, and thus Edith ordinarily uses the more elaborate FLYING SAUCER grid. The 16 three-letter words are again printed on cards. Here there are three disjoint parts to the misgraph. The three sets are F-N-G-C, I-A-U-E and L-Y-S-R. They may be recalled by using the stylized Flying Saucer label provided below.

![FLYING SAUCER](image)
The trick works the same as before. Mark gives, say FEY, LEG, and ERN (from a stagger, but Edith does not need to know this), and Edith uses F-N-G-? to obtain ?=C, Y-L-R-? to get ?=S, and E-E-E-? to get ?=E. Hence the unknown is SEC.

FLYING SAUCER

The MOUSTERIAN square could also be used in a prediction trick, but instead we proceed to the more baffling EDITH CAMUS-BROWN 5x5 square. The 15 letters in the misgraph are in three disjoint sets: A-E-I-O-U, W-R-C-D-S and M-H-B-T-N, superposed in the pentagonal misgraph diagram on the next page. It is quite important not to disturb the cyclic order of the letters because the distribution for a stagger will always display some sort of symmetry with respect to the pentagon. The symmetry may sometimes be too ambiguous for us to be able to identify a quad so we will only use staggers. We illustrate with an example.
Suppose Mark gives Edith SOB-HOW-SHE-SIN from a stagger. The missing vowel comes from O-O-E-I-? and only ?=E preserves symmetry. Similarly, B-H-H-N-? yields ?=B or N, and the combination W-S-S-S-? can only mean ?=D. BED is the only word from the word list that works as the unknown.

The attentive reader may have noticed that each of our word squares is actually an instance of a set of Graeco-Latin squares. Briefly, an nxn Latin square is one where every one of n letters occurs exactly once in every row and every column. If we have two such squares, one in Latin and one in Greek, and if we can superpose the two so that every possible pair of Latin and Greek letters occurs exactly once, we have a Graeco-Latin square. Graeco-Latin squares are also called orthogonal or Euler squares.

In EDITH CAMUS-BROWN, we have three mutually orthogonal squares and could possibly have a fourth if we could add five new letters to take the places of the numbers 1,2,3,4 and 5 in the grid. It would then be possible to add quads to our prediction tricks. For example, if Mark keys the corner CON-WIT-HUD, Edith confuses MEW and BAW for the unknown corner. If the numbers are also present, she would know that MEW 3 could not be present in the same row as CON 3 or column as HUD 3. Therefore, the only answer is BAW 5. Perhaps some reader can find 25 four-letter words using 25 letters or the alphabet so that the ultimate word 5x5 can be constructed.

No pair of orthogonal squares exists for the 6x6 case, as was proven in 1900 by G. Terry, confirming a conjecture of L. Euler from 1782. However, nxn squares for n=7,8,9,10 and 11 do exist and could probably be constructed using letters.

And by no means are squares the only possible shapes. ASTEROID 12 and ASTEROID 16, on the next page, are examples of four-in-a-row on triangles, each with constant ASTEROID. The four circles all spell FLYING SAUCER along their circumferences.
"BLACKY" SPIDER is a wonderful (i.e., hard) puzzle best played by printing the two-letter words on scrub tiles and trying to arrange them so that each of the three hexagons and each of the three ribs transpose into the constant BLACKY SPIDER (see diagram on next page). It is also possible to look carefully at the misgraph and choose numerical values so that the integers from 0 to 17 can be placed on the nodes to sum to a magic 51.
Even though 6x6 Euler squares do not exist, Edith’s friend NORWICH BUMSTEAD has devised the next best possible square. No letter is repeated in any row, column or main diagonal, and any two letters appear together at most once in the grid. From the set A-E-I-O-U-S each letter appears six times while the remaining nine letters occur exactly four times each.

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<thead>
<tr>
<th>BA</th>
<th>ED</th>
<th>IT</th>
<th>OW</th>
<th>MU</th>
<th>CS</th>
</tr>
</thead>
<tbody>
<tr>
<td>WE</td>
<td>OH</td>
<td>RA</td>
<td>SB</td>
<td>ID</td>
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<td>TO</td>
<td>ME</td>
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NORWICH BUMSTEAD
The pentacle, or five-pointed star, has been a mystical symbol for centuries but can not be made magic on the five lines by any placement of the integers from 1 to 10 (see Martin Gardner's *Mathematical Carnival* for details). Nor can we make it magic with words. Therefore, we seek to make it antimagic everywhere, and the MOUSTERIAN star accomplishes this feat. Antimagic means when we transpose among sets, we always get different words. The sets we have in mind are the five lines, the five small triangles (e.g., RE-IN-MS), and the five large triangles (e.g., RE-SO-UT).

Here are the transposed words with mousterian word complements:

Lines: SENORITA (MU), MINOTAUR (ES), MOISTURE (AN), SUMERIAN (TO), SEAMOUNT (IR)
Small Triangles: MINERS (AUTO), RIMOUS (NEAT), TUMION (SEAR), MANTOS (URIE, a town in Wyoming), EOSIAN pertaining to EOS (TURM)
Large Triangles: MEATUS (IRON), ARIETO Latin "to ram" (SNUM), MANURE (OTIS), ROUTES (MAIN), NUTRIA (SOME)

Of course, the star uses two copies of MOUSTERIAN, but no letter is ever repeated in any word.

Edith has devised another kind of prediction trick based on the star ST. WORDBUCH, MAINE whose misgraph is the RUNE shown on the next page. Mark is asked to arrange scrub tiles of the words on the ten points so that no two words with a common letter abut. Since the rune is the graph of hits and is the famous Petersen graph, it would be quite impossible to place the scrubs to abut common letters, since the Petersen graph is non-Hamiltonian. Although there are 120 ways of placing them to non-abut that at least look different, it will still take some time for Mark to complete the puzzle. When he finally succeeds, he turns all the scrubs over. Edith knows that on the star a given word's three letters will be repeated on a triangle in the star. The triangle will lie on no common lines with the given word. For instance, the word RED is on no lines with the triangle CAR-HEN-DIM--we say that RED can not see the triangle. Similarly, HUT does not see the triangle HEN-CUB-WIT. Edith asks Mark to turn over any two scrubs. Suppose he exposes RED and MOB. From the rune, RED and MOB both connect DIM. Therefore, DIM does not see...
the triangle RED-MOB-? This allows Edith to find and point to DIM and name it, and to point to ? and name it correctly as WIT. If Mark chose RED and HEN to uncover instead, Edith knows that RED can not see the triangle with HEN as a vertex and therefore the two unknown vertices must be, in some order, CAR and DIM. She simultaneously turns both over, naming the two as she does so. The reader will discover that with practice he can name virtually all of the nodes.

THE RUNE OF
ST. WORDBUCH, MAINE
As a last prediction trick, we employ the services of Edith's friend, the village dowager INA CHOWBLY DUMPSTER. The nine words BODE, BUMP, CARD, CHEW, HYMN, LOUT, SAIL, SPRY and TWIN are written on cards. As a puzzle, Mark is to arrange these in a 3x3 grid so that no row or column contains duplicated letters. There are 72 different solutions to this puzzle and sooner or later Mark finds one. The cards are turned down and a further mix is performed, if desired, by interchanging any two rows or two columns. Edith, whose back has been turned all along, now asks Mark to select a quad and turns up three key cards in it. Suppose they are TWIN-BODE-SAIL. Edith quickly predicts that the fourth member of the quad is BUMP and it turns out to be so.

This is how Edith does it. She memorizes the scheme

<table>
<thead>
<tr>
<th>PLY</th>
<th>MRS</th>
<th>ICD</th>
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</thead>
<tbody>
<tr>
<td>WHO</td>
<td>ATE</td>
<td>BUN</td>
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</table>

Exactly two of the key words will have a letter in common, here the I in TWIN and SAIL. The letter I is paired in the scheme with B. The missing word will be the other word with B (hence, BUMP).

A variation of this trick can be done by asking Mark to arrange the nine cards so that every row and column contain words that all have letters in common. The scheme can still be used to predict in a quad as the reader will be able to discover for himself.

The construction of thematic magic squares can be a nice diversion. As an example use the chemical element ARSENIC and "burn" C from it. The remaining six letters can form the 3x3 magic square whose rows are the chemical symbols AS-ER-NI, NE-SI-RA and IR-NA-SE. Another square with rows FA-HE-IT, ET-IF-AH and HI-AT-FE is magic with constant the name FAITH E. Make a puzzle for your friends out of these and then amaze them with a quad prediction trick!