THE ELUCIDATORS

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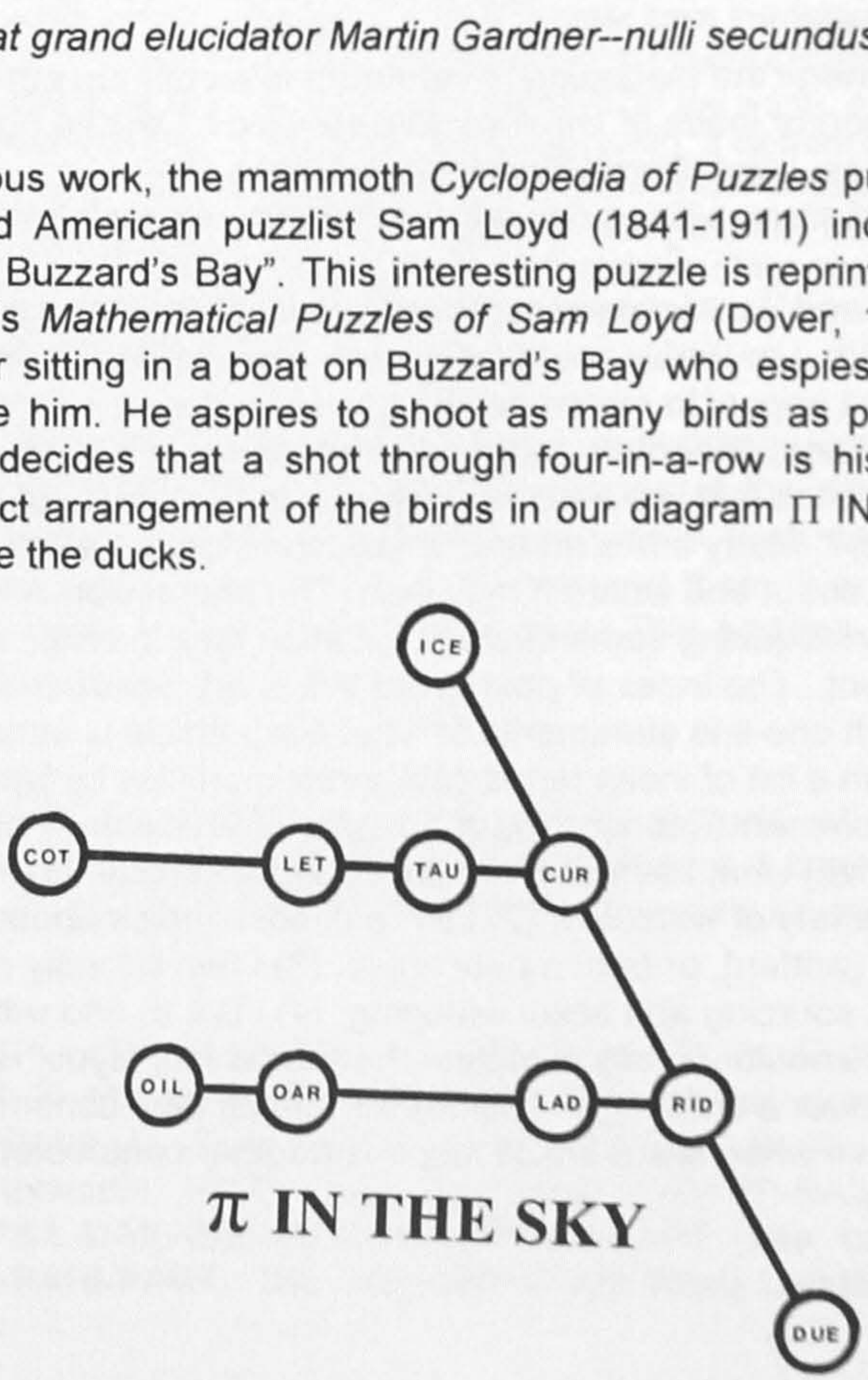
Dedicated to that grand elucidator Martin Gardner--nulli secundus

In his posthumous work, the mammoth *Cyclopedia of Puzzles* published in 1914 by his son, the noted American puzzlist Sam Loyd (1841-1911) includes the problem "Duck Shooting at Buzzard's Bay". This interesting puzzle is reprinted as problem #96 in Martin Gardner's *Mathematical Puzzles of Sam Loyd* (Dover, 1959). It involves a rather inept hunter sitting in a boat on Buzzard's Bay who espies a formation of ten ducks flying above him. He aspires to shoot as many birds as possible before they flutter away, and decides that a shot through four-in-a-row is his best chance. We reproduce the exact arrangement of the birds in our diagram Π IN THE SKY and, for convenience, name the ducks.



The hunter's first shot misses completely and in the subsequent confusion the ducks fly away, still in rows of four. The puzzle is: "By changing the position of the fewest possible number of the ten ducks, arrange them so there will be five rows of four in

each row." Before we give the solution, the reader may wish to try his hand at the puzzle by placing ten small tokens on our diagram and moving the tokens around.

Loyd reused the Buzzard Bay puzzle in *Cyclopedia* on page 194, as "A Clever Coin Trick". He arranges ten coins exactly as in our Π IN THE SKY and asks for the fewest moves so that there will be five rows of four. In his second collection of Loyd puzzles, *More Mathematical Puzzles of Sam Loyd* (Dover, 1960), Gardner reproduces the similar "A Nautical Problem" (#59) where ten ships are arranged in two rows of five and one is asked how to make five rows with four ships in each row by changing the positions of exactly four ships.

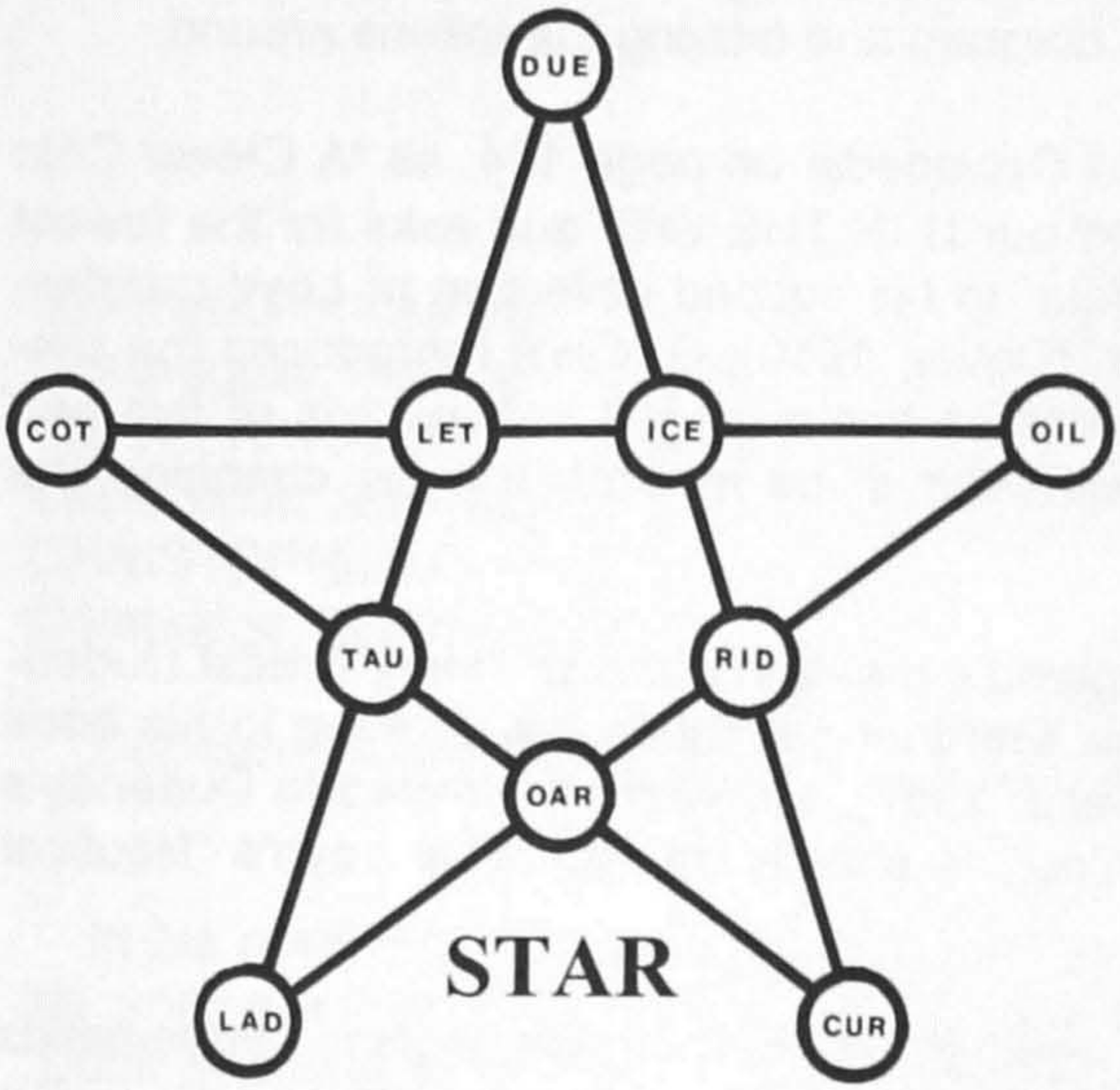
Meanwhile, another elucidator and England's greatest puzzlist, Henry Ernest Dudeney (1857-1930), produced similar puzzles. Gardner describes two of them in his book *536 Puzzles & Curious Problems* (Scribner's, 1967), a collection devoted to Dudeney's puzzles. One, #439, "Deploying Battleships", is exactly the same as Loyd's "Nautical Problem".

Dudeney used the Battleship problem again in his book *Amusements in Mathematics* (T. Nelson, 1917), as problem #210 "The Ten Coins", which we reproduce below. His solution to this puzzle includes answers complete enough to solve any similar puzzles. He says these puzzles "...are variations of the case of ten points arranged to five lines of four...There are six fundamental solutions, and no more, as shown in the six diagrams. These for the sake of convenience, I named some years ago the Star, the Dart, the Compasses, the Funnel, the Scissors, and the Nail."

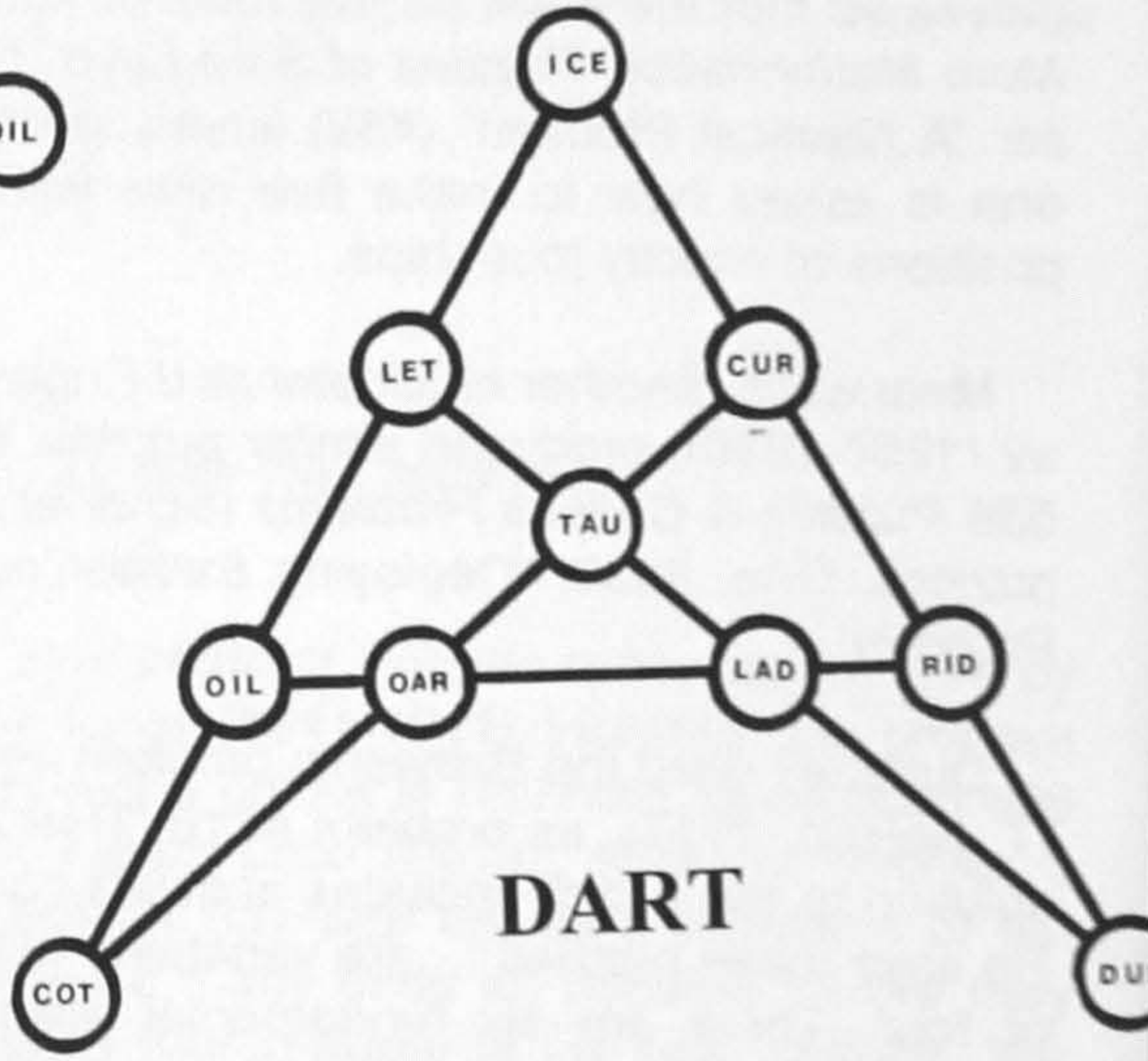
THE TEN COINS

We agree with Dudeney that there are precisely six solutions to the five lines of four problems in general and give his graphs with our labels below. In addition, we include the modern MOON LANDER to make a point about certain puzzles and games we will shortly introduce.

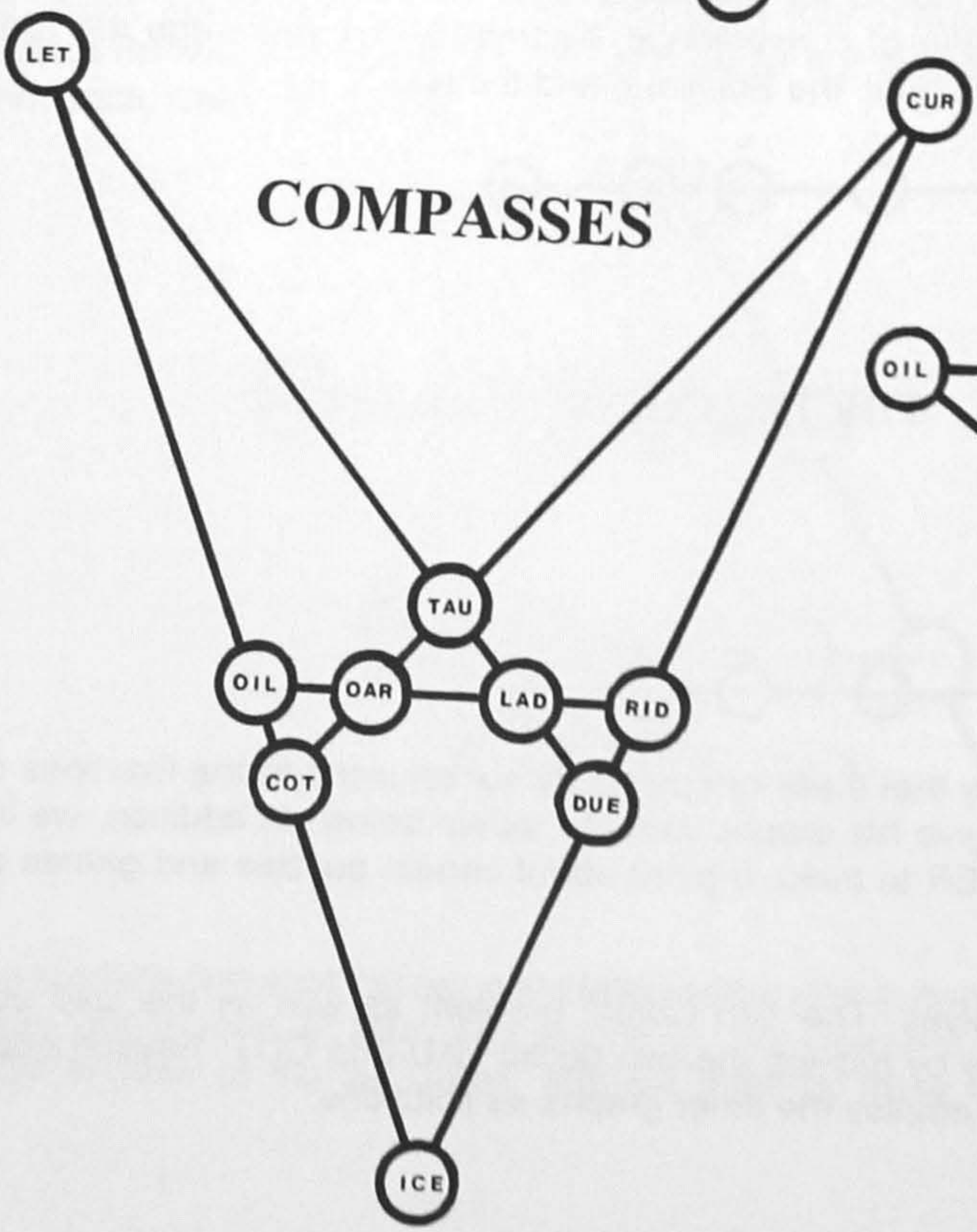
Dudeney's Scissors solves "The Ten Coins" problem as well as the ship puzzles. Dart solves Buzzard's Bay by moving the two ducks TAU and COT. Several additional problems in *Amusements* employ the other graphs as solutions.



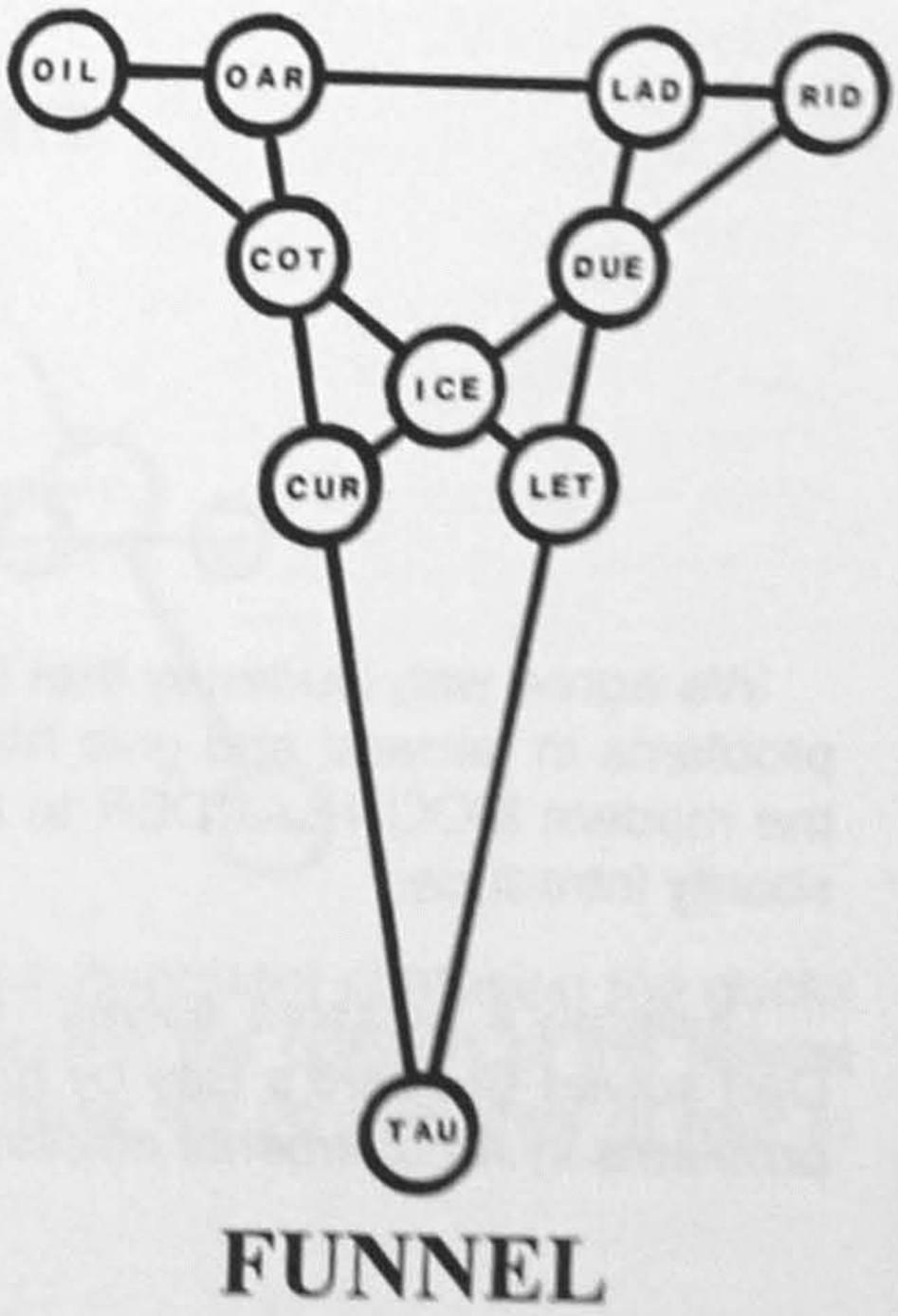
STAR



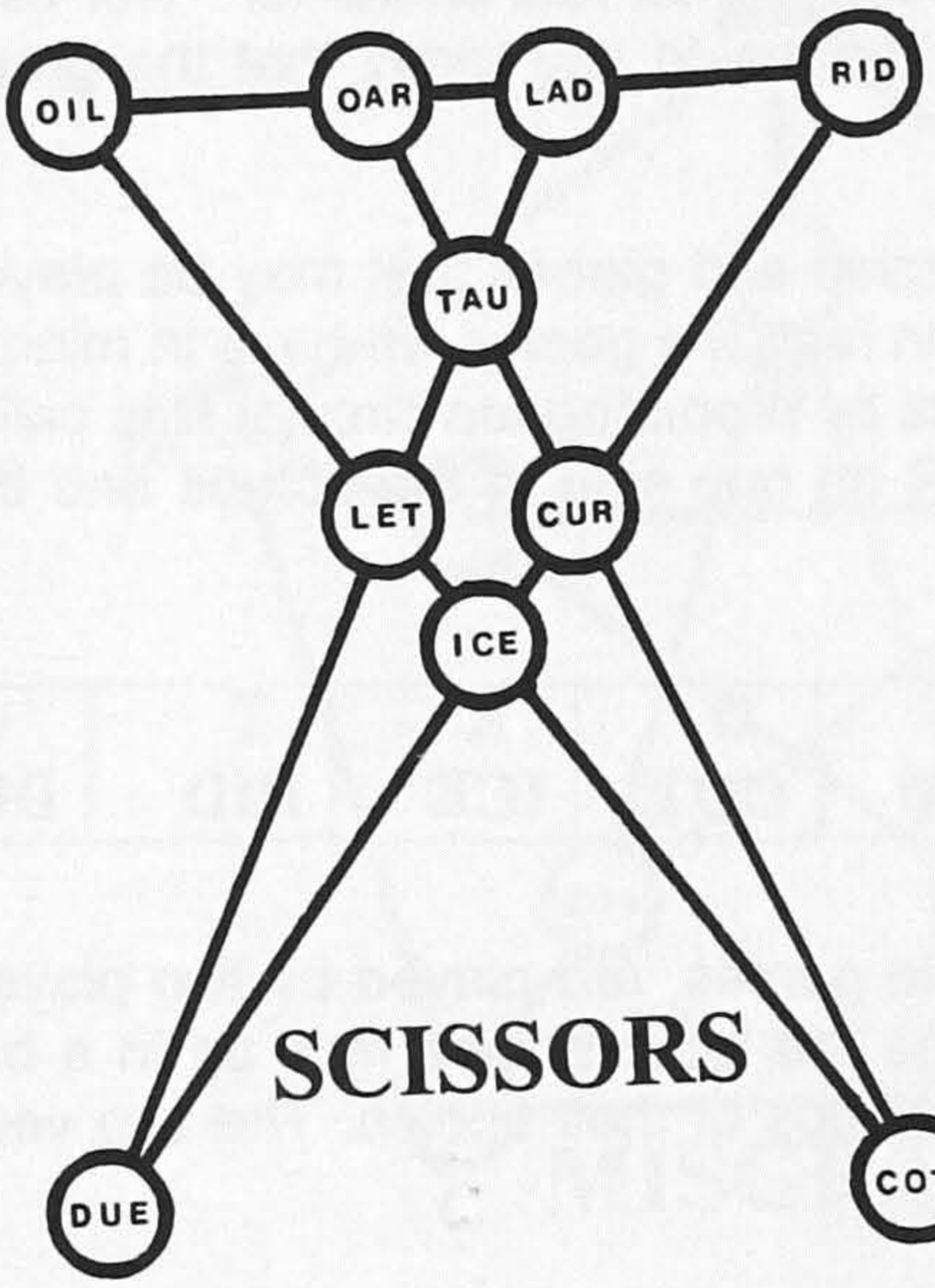
DART



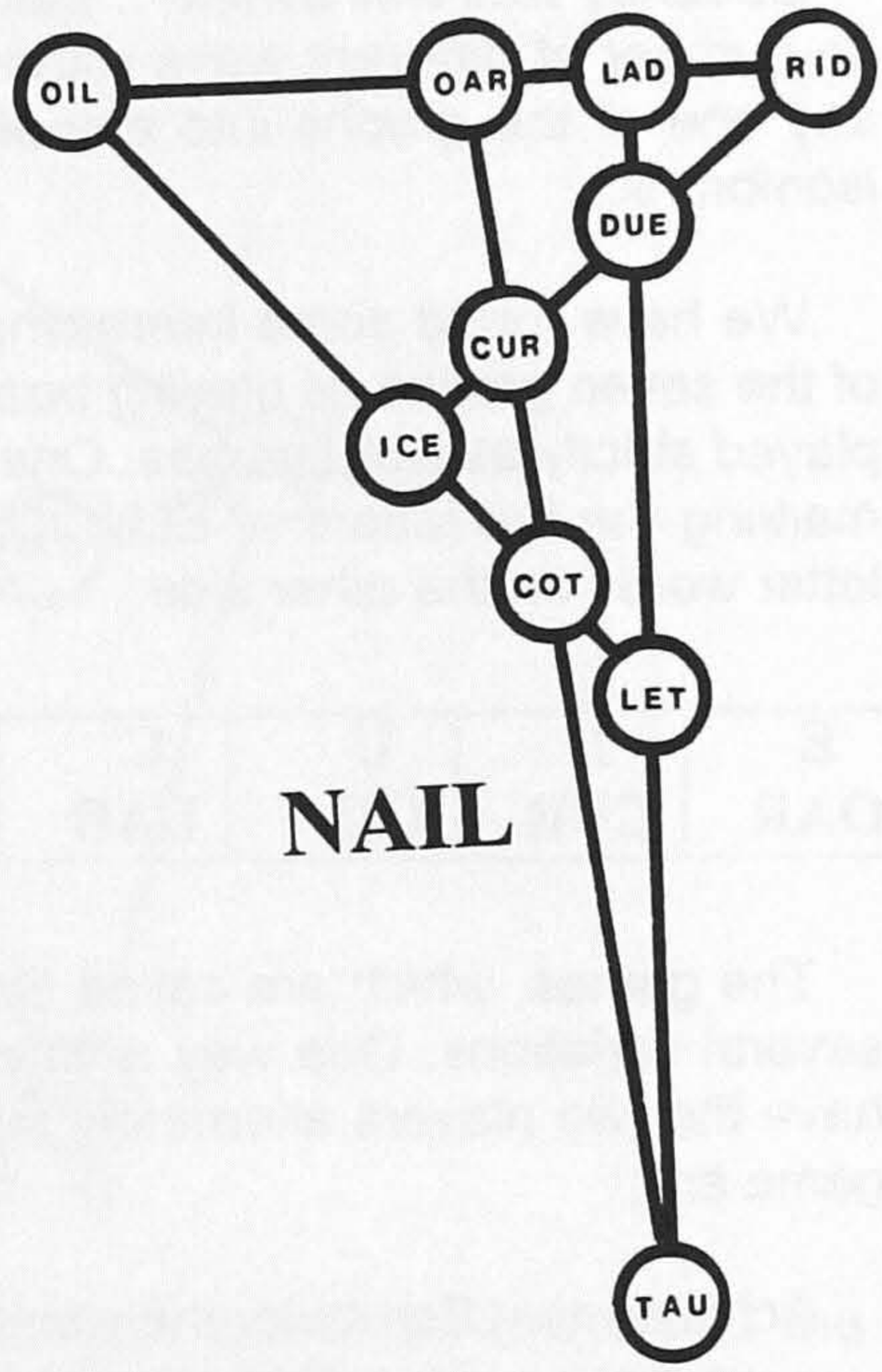
COMPASSES



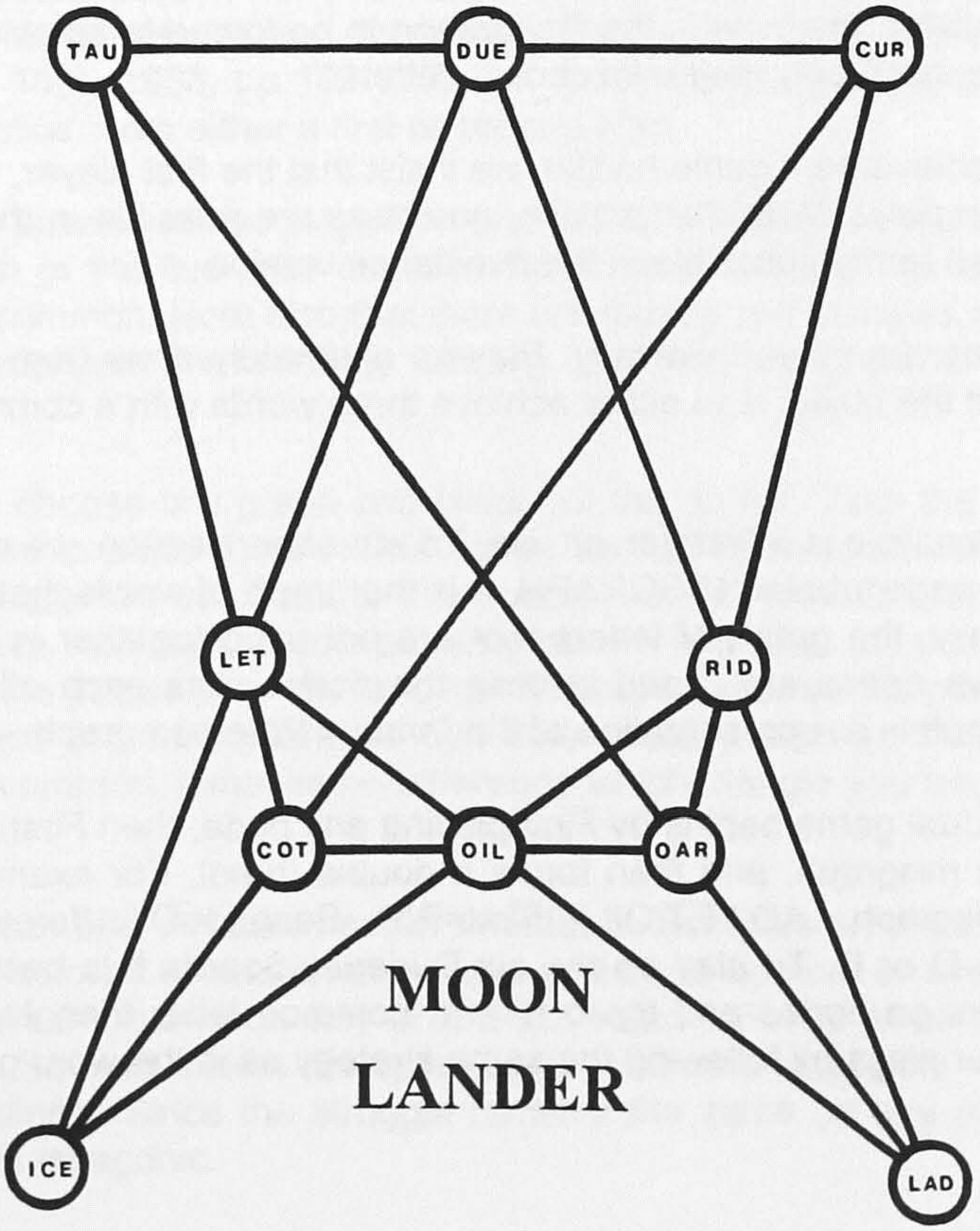
FUNNEL



SCISSORS



NAIL



MOON LANDER

Dudeney was well aware "...that any one of these forms may be distorted in an infinite number of different ways without destroying its real character." Nor can one distort any one of the graphs into any other. We would say today that the graphs are non-isomorphic.

We have found some interesting puzzles and games that may be played using any of the seven graphs as playing boards. In fact, the games we have in mind may also be played strictly as word games. One starts by preparing ten circular tiles called scrubs by marking the ten letters of ELUCIDATOR on one side of the scrubs and the ten three-letter words on the other side.

E OAR	L CUR	U OIL	C LAD	I TAU	D COT	A ICE	T RID	O DUE	R LET

The games, which are called Scrubble games, are played by two players and have several variations. One way is to arrange the letter scrubs face up in a bone pile and have the two players alternately select scrubs of their choice. The two versions of this game are:

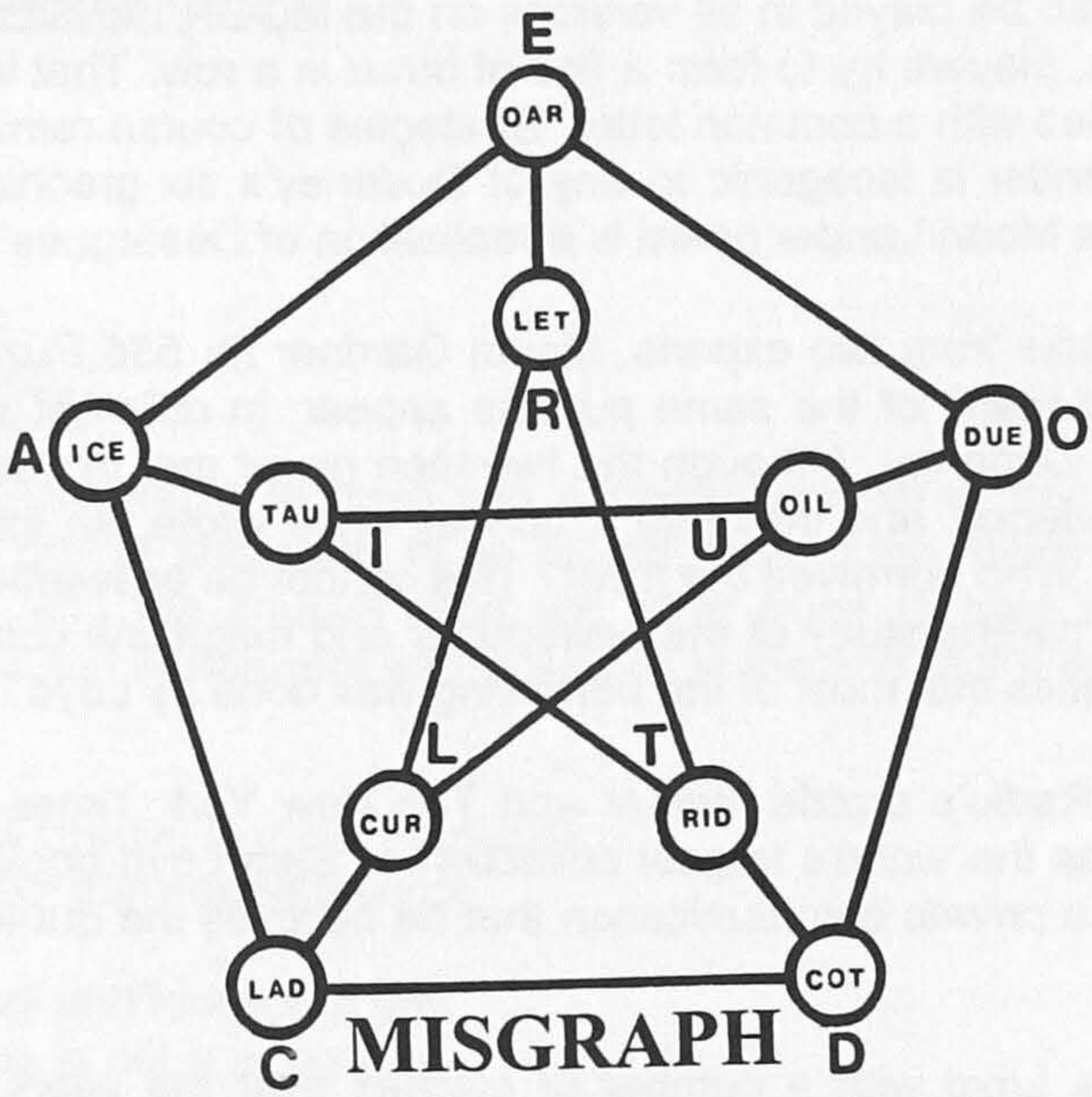
Achievement Scrubble: the winner is the first person to complete one of the ten words using only their own chosen letters
Avoidance Scrubble: the loser is the first person to be forced to complete one of the ten words using only their own chosen letters

To make the achievement game harder, we insist that the first player, in order to win, must do so in four plays. With this proviso, no draws are possible in the achievement version while draws remain possible in the avoidance version.

Two dual games are played similarly. Players alternately draw from a bone pile of face up words and the object is to either achieve three words with a common letter or to avoid it.

Achievement Scrubble is a first person win. To win either version, we employ the diagram on the next page labeled MISGRAPH. It is the graph of words that have no common letter or, dually, the graph of letters that are not used together in any of the ten words. That is, two nodes are joined in case the nodes miss each other. It is worth noting that this graph is a representation of the famous Petersen graph.

The win in the dual game begins by First playing any node, then First forces Second to play in his own misgraph, and then forms a double threat. For example, First=ICE, Second=CUR (misgraph LAD,LET,OIL), First=RID, Second=OIL (forced), First=DUE and threatens the D or E. To play on the six Dudeney boards it is best to use transparent plastic disks on nodes and try to form a common letter triangle of their color. First can win in four plays by following the same strategy as in the word game version.



Avoidance Scrubble is more challenging, and we must leave the analysis to the reader. It is a second person win when expertly played (see the Desargues diagram in Frank Harary's "Achievement and Avoidance Games on Finite Configurations," J. of Rec. Math., Vol 16(3) 1983, pp 182-187), but there are many traps, and good players often win in practice, from either a first or second start.

Our Scrubble puzzles will be played on any one of Dudeney's six graphs. Notice that on a graph each of the five lines contains four words and any two of the four words have a letter in common. Note also that there are exactly ten triangles in each of the six graphs. These triangles are found by locating the three occurrences of each letter of ELUCIDATOR.

For a puzzle, choose any graph and black out the nodes. Take the ten word-scrubs and ask someone to place them on the nodes so that abutting scrubs have a letter in common. There are 120 solutions to this puzzle that will look different to the eye. All solutions will have the four-in-a-row property but we need not tell our victim this. Most people will find the puzzle very challenging. When you attempt the puzzle, you will find it easiest to place the letter triangles--this automatically assures that abutting scrubs have a letter in common. It makes no difference which triangle you begin with; they are interchangeable.

Two players can play Achievement or Avoidance Scrubble on the blacked-out board too. They place colored tokens on the nodes and try to achieve or avoid triangles. The strategies remain the same on any of the six boards. For example, in Achievement Scrubble, when Second makes his opening play, visually locate his misgraph triangle and play accordingly. Since the struggle remains the same on any of the boards, we call these boards isoagonic.

Scrubble may also be played in all versions on the MOON LANDER diagram where, instead of triangles, players try to form a line of three in a row. That is, each of the ten lines has three nodes with a common letter. Strategies of course remain essentially the same, so Moon Lander is isoagonic to any of Dudeney's six graphs. Mathematicians will observe that the Moon Lander board is a realization of Desargues' theorem.

Some final remarks from two experts. Martin Gardner (in *536 Puzzles*) says "Readers...will notice that many of the same puzzles appear, in different story forms in the books of Loyd and Dudeney. Although the two men never met in person, they were in frequent correspondence, and they had, Dudeney once wrote, an informal agreement to exchange ideas. Who borrowed the most? This cannot be answered with finality until someone makes a careful study of the newspaper and magazine contributions of both men, but it is my guess that most of the borrowing was done by Loyd."

National Public Radio's puzzle master and The New York Times crossword editor Will Shortz, who has the world's largest collection of Sam Loyd books and ephemera, has informed us in a private communication that he believes the quote from Gardner is fair.

"Dudeney credits Loyd with a number of puzzles over the years, but I can't ever remember Loyd crediting Dudeney," said Shortz, "so of the two, Dudeney would appear to be the more honest."

Shortz also recalled that Dudeney's wife, Alice, who was a rather famous novelist at the turn of the century, kept a journal in which she reported her husband's anger over Loyd's theft of his puzzle ideas.

"Someday I hope to help solve this mystery," Shortz continued, "by comparing the first publication dates of similar Loyd and Dudeney puzzles. For example, I've traced 'Duck Shooting at Buzzard's Bay' back to a Loyd newspaper feature in 1899. Other similar versions appeared in 1900, 1903, and later. From extensive research over the past 15 years, I've compiled an index of the great majority of Loyd's puzzles in the *Cyclopedia* (plus thousands of others that didn't make it into that volume). If someone would do a similar index for Dudeney, we could compare!"