PLANET PACKING

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Suppose that one has a list of words. What is the smallest string of letters needed to exhibit all of these words, each spelled out in order but not necessarily using adjacent letters in the string? For example, **one, two** and **three** can be embedded in the eight-letter string **thrwonee** (or several others), but no seven-letter string is possible. It is easy to see that seven is the theoretical minimum, for six different letters appear once in any word, and one letter appears twice in a word. For longer lists, however, letter counts of individual words yield only a lower bound for the minimum achievable string length.

To construct a minimum-length string, one needs an algorithm—a set of rules that specify how list members are to be added one at a time to an evolving string. One part of the algorithm must specify which list member is to be added next, and another part must specify exactly where it is to be added. There is no guarantee that any algorithm will end up with a minimum string, but the quality of the algorithm can be compared with ones in which list members are added at random.

This article presents a plausible algorithm. To fix ideas, consider the list of the nine planets, which yields a lower bound of 21 letters (chijlovy each appear only once in the list, ampst appear at most once in any word, and enur appear twice in at least one word). First remove the once-only letters which can readily be inserted in the string at the end, creating the list merur, enus, eart, mars, upter, saturn, uranus, neptune and put.

The more letters a pair of planets have in common in the same order, the shorter the string containing both, so the first task is to identify such pairs. Only three pairs have three letters in the same order: URaNus-satURN, nepTUNe-saTUrN and uraNUS-veNUS. The algorithm chooses any pair to start the string, and then adds the other two planets in either order. URaNus-satURN yields the string **saturanus.** Adding enus, we obtain **(e,sat)Uranus** where the parentheses enclose two strings that can be interleaved in any order. When neptune is added, four of its letters (etun) can be accommodated in order by modifying the interleaved **(e,sat)** so that E comes before T: **(sa, nep)turan(us,e).** This sequence accommodates three letters of mARS and mErUR, or three letters of EArT if A comes after E in the first interleaving. Adding them in any order, we end up with the string **(m,n)e(sar,p)turan(us,e)**. This leaves only the planets upter and put, both somewhat recalcitrant because the string contains only the ptu order, so the final string becomes **(m,n)e(sar,pup)turan(us,er)**. To this string of 18 one must add the 8 single-occurrence letters, for a string of 26. Can anyone devise a better algorithm to lower this to 25?

Readers might like to find the minimum string for the colors of the rainbow (red, orange, yellow, green, blue, indigo and violet), the days of the week, the months of the year, the names of the Zodiac, or the names of the Greek letters. For longer lists such as presidential surnames or state names, any reasonable algorithm almost certainly requires the aid of a computer.