

DUAL PERMS

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This article responds to the challenge set by Hugo Brandt Corstius in the May 2001 issue of Word Ways. He sought pairs of words that contained the same letters (i.e., permutations of each other), which were dual in the sense that a position occupied by a vowel in one was occupied by a consonant in the other. This article takes vowel to mean one of A, E, I, O or U.

He mentions that such pairs of words can only be of even length. If the first word contains v vowels, then its partner must contain the same v vowels because it is a permutation, and v consonants because it is a dual. So both contain $2v$ letters. By a similar argument, both contain $2c$ letters. Therefore $v = c$. Each has the same number of vowels as consonants, hence an even number of letters.

It transpires that there are plenty of such pairs of words of lengths 4, 6, and 8, with a few of length 10, and none longer. To reduce the numbers further, it is possible to select just pairs in which one word is the reverse of the other. Independently, it is possible to find pairs in which each word is a heterogram, i.e. with no two letters the same. In the case of lengths 4 and 6, it is possible to find pairs that comply with both the reversal and the heterogrammatic restrictions.

Readers will recall the large number of words that can be made from such combinations of letters as AELRST or AEGINRST (see my article "The Commonest Alphomes" in the May 1999 Word Ways); if we have 5 such words, there are $4+3+2+1$ possible pairs, and each qualifying pair is included in the counts below. Sometimes a word may pair with more than one other, so might be called an amenable or fecund dual. Although such multiples may be found at a number of lengths, some analysis is given under 4-letter words later in the article.

The following table will be explained by the example immediately following.

col: 0	1	2	3	4	5	6	7
Word length	Dual pairs	Alphomes with at least one dual pair	Max dual pairs from single alphome	Total reversal pairs	Heterogrammatic alphomes with at least one dual pair	Heterogrammatic alphomes with at least one reversal pair	Max reversal pairs in single alphome
4	3656	980	32	671	662	329	*5
6	2819	1330	27	75	577	20	1
8	362	299	5	0	63	0	0
10	36	35	2	0	3	0	0

* max of 5 occurs twice. All other maxima in columns 3 and 7 occur once only.

EVIL-LIVE (r), EVIL-LEVI, EVIL-VILE, LIVE-IVEL, LEVI-IVEL (r), VILE-IVEL

In this shortened example (another set of letters also fits), the same set of letters (alphome) gives rise to 6 duals, so adds six to column 1 and one to column 2. The two reversal pairs, marked (r), add two to column 4. The alphome is heterogrammatic, so one is added to column 5. Because there is at least one reversal pair, and the alphome is heterogrammatic, one is added to column 6. No records are set by this example, so columns 3 and 7 are not changed.

Readers may recall that the number of words that can be made into another word when their letters are rearranged falls sharply from 88% at length 4 to less than 10% at length 10. Furthermore, of the number of words of a given length that will permute into at least one other word, the percentage of such permutations which are duals falls dramatically from 59%, through 11% and 2%, to ½% as the word length increases from 4 to 10 (in steps of 2). At the same time, the number of heterograms falls from well over 76% to less than 6% (see my article "How Many Anagrams Are There?" in the November 2000 Word Ways), more than countering the much larger number of words available. These facts help to explain the scarcity in this article of longer examples.

Words are taken from the OED and Webster 2 except where noted. Ch = Chambers, EDD = English Dialect Dictionary, F&W = Funk & Wagnalls New Standard Dictionary, CL = Columbia-Lippincott Gazetteer of the World, MED = medical term (from any of various dictionaries), OSPD = Official Scrabble Players Dictionary, pname = personal name, Pull = Pulliam, q = quotation in OED, Sted = Stedman's Medical Dictionary, TEA = The Electronic Alveary, vf = OED variant form, WNB = Evelyn Wells *What To Name The Baby* (1953). Note that the source of a word is usually only quoted the first time the word appears.

Fecund 10-letter words

The most fecund word is ELUCIDATED, with the two partners DELUCIDATE and DELICATUDE. The theoretical maximum, for a heterogram, is 120^2 partners. These two pairs are from the most fecund alphome. The string ELUCIDATE illustrates a method of finding some examples: take an alternating sequence of vowels and consonants, shift the sequence by an odd number of letters, then try to fill the spaces in both sequences to make words. Heterogrammatic examples are in the minority: RADICULOSE—ORACULISED and the following AEIOU pairs from Nomenclator Zoologicus: ISOPETALUM—MESOTIPULA, and ELAPOTINUS—NESOTIPUILA. I found no reversals.

Other examples include ELAPIDATED—DELAPIDATE, ISOCLASITE—SOCIALITES, and ENUMERATED--DENUMERATE.

Fecund 8-letter alphomes

At this length, there are a sufficient number of examples that we should be able to find a few pairs with aptly related words.

The most fecund alphome produces five pairs:

ANORETIC—CONARITE, ANORETIC—CERATONI (TEA), ANEROTIC—CONARITE, ANEROTIC—CERATONI and CORTAINE—ACOINTER (Old Fr, whence acquaint).

This is also a heterogram.

Runners-up for fecund alphome are:

CINERAMA—AMERICAN, AMACRINE—RECAINAM (Sted), AMERICAN—CAMERINA (WW76-100), AMERICAN—CAMARINE

ACTINIDE—DIACETIN, ACTINIDE—DIANETIC, INDICATE—DIACETIN, INDICATE—

DIANETIC.

AGE LIMIT—LIMITAGE, AGE LIMIT—LEGITIMA (Latin = lawful), IMAGILET—LIMITAGE, IMAGILET—LEGITIMA

AGONISER—ROSAGINE, AGONISER—GENISARO, ORIGANES (OED origin)—ROSAGINE, ORIGANES—GENISARO.

Another runner-up is ANIMATER, as below.

Fecund 8-letter words

The most fecund word is ANIMATER, with the four partners MARANITE, MARINATE, TAMARINE, and MARITANE (obs. Maritime). Thus the most fecund word is not a heterogram, which in theory could give as many as 24^2 partners.

Some other pairs in heterogrammatic alphomes are:

Single pairs: MEXICANO—ANOXEMIC, MOISTURE—OSMIURET, OBVIATES—BOASTIVE, UNMAILED—MAUDLINE (vf)

Double pairs: URINATED—DATURINE and INDURATE—RUINATED, ISOLATED—SOLIDATE and ISOLATED—SODALITE, RUINATES—UNSATIRE and UNSATIRE—TAURINES

Triple pair: ELASIPOD—LAPIDOSE and EPISODAL—LAPIDOSE and LAPIDOSE—OPALISED

Quad pair: AGONISER—ROSAGINE and AGONISER—GENISARO and ORIGANES—ROSAGINE and ORIGANES—GENISARO

Other examples include: COMENCI (perfect gymnast)—ECOMANIC, ICE-WATER—WATER-ICE (which illustrates a second method of finding examples, i.e., compound nouns, as does also OVERTAKE—TAKE-OVER), ESTIMATE—TEATIMES, ISLAMITE—SEA LIMIT, EBONISED—SIDEBONE, UTRICIDE—DIURETIC, DEVOLUTE—EVOLUTED, and the three related heterogrammatic pairs ATOMISED—SODAMITE, ATOMIZER—MAZORITE (vf) and ATOMIZES—SOMATIZE (Stedman).

I found no reversals.

Fecund 6-letter alphomes

The most fecund alphome is a heterogram, and produces 27 pairs:

MOANER—ANMORE (Hampshire)	ANMORE—EANORN	NOMARE (vf)—AMONER (vf)
NOMARE—ANOMER (Pull)	NOMARE—ENAMOR	NOMARE—OREMAN
MONERA—AMONER	MONERA—ANOMER	MONERA—ENAMOR
MONERA—OREMAN	AMONER—ROMANE (vf)	AMONER—NAMORE
AMONER—MARONE (vf)	AMONER—MORENA	NORMAE (Sted)—ONEARM
ANOMER—ROMANE	ANOMER—NAMORE	ANOMER—MARONE
ANOMER—MORENA	ROMANE—ENAMOR (r)	ROMANE—OREMAN
ENAMOR—NAMORE	ENAMOR—MARONE	ENAMOR—MORENA
NAMORE—OREMAN	MARONE—OREMAN	MORENA—OREMAN.

The runner up is also a heterogram, contains SATIRE, and has 19 pairs, including the reverse pair STRIAE—EAIRTS (EDD).

Fecund 6-letter words

The most fecund word is the heterogram IMARET, with the seven partners RIMATE, MERITA (Ch. Latin terra merita, whence turmeric), METARI (Ch. Latin castra metari, whence castramentation), MIRATE, MERATI (TEA), MATIRE, and MARITE. The runner-up is ISLARE (obs. Ashlar), with its six partners LEISAR (obs. Leisure), RIALES (Web3 under real) RAILES (vf), LAISER (vf), SAILER, and REALIS (Ch. Latin, whence real). This seven compares with a theoretical maximum of $6^2 = 36$ partners for a heterogram.

Heterogrammatic pairs which are also reversals (in addition to the two above):

DECIMA—AMICED	CINEMA—AMENIC (Sted)	
ERICAS (OSPD)—SACIRE (Ch, Low Latin root of seize)		DEGAMI (OSPD)—IMAGED
DIAKNE (vf)—ENKAID (TEA)	DELIMA (Sted)—AMILED (vf)	SEMINA—ANIMES (both OSPD)
NAVIRE—ERIVAN (OED Sophy, the ancient seat of the sophis)		
AMORES (Latin pl love, works by Ovid and D H Lawrence)—SEROMA (Sted)		
ILOPAN (?)—NAPOLI (Napoli di Romania, Italian port)		UTINAM—MANITU
LIBERO—OREBIL (vf)	ECITON (ant, OED under forager)—NOTICE	
NO-SIDE (Ch, Rugby)—EDISON (Thomas Alva)		UNITED—DETINU (vf)
NEROLI—ILOREN	ELUTOR—ROTULE	

At length 6, there are over 1000 alphomes, and, very roughly, these are split half-and-half into those which are heterograms and those which are not. There is therefore little merit in listing heterograms by themselves. Reversals are far less common, so a small selection of these follows (they have at least one repeated letter, else they would have been listed above):

TARARA—ARARAT (Mount)	APPEAL—LAEPPA (vf)	LAMINA—ANIMAL
ASOMAS (Sted)—SAMOSA	SIBILA (vf)—ALIBIS	

Here is a selection of other pairs, some heterograms, some not, in alphomic order:

BAZAAR—AZABRA (vf)	MAARAD (Pull)—ARMADA	CARIBA (vf)—ARABIC
LABIAL—ABILLA	CAESAR—ASCARE/ACRASE (vf)	CASSIA—ISAACS
ASCULA—CAUSAL	GADEAN—AGENDA	ANDEAN—DEANNA (female climber)
AGLARE—LAAGER	AGEMAN—MANAGE	SAVAGE—AGAVES (OSPD)
AWAKEN—WAKANE (vf)	ALETAP—PALATE	PEASAN (vf)—ESPANA
MANILA—ANIMAL	MALAWI—AWALIM	AVAILS—SALVIA
KORANA—ANORAK	ALUMNA—MANUAL	PAUSAL—UPSALA
BEARDE (vf)—ABREED (Pull)	AIRBED—BRAIDE (vf)	ABSIDE (vf)—BIASED
DIBASE—ABIDES	ABDITE—BAITED	ENABLE—BALEEN
BASILE (vf)—ISABEL	BULEAN (vf)—UNABLE	OBLATE—BOATEL
BUTANE—ABUTEN	BOATER—ORBATE	RIBOSA (vf)—ISOBAR
ACETIC—CATICE	ACCUSE—CAECUS (Sted)	DEACON—ACNODE
ALERCE—CEREAL	PEACES—ESCAPE	FAUCET—UTFACE
MALICE—AMELIC (Sted)	ECLAIR—CAIRLE (vf)	CINEMA—ICEMAN/ANEMIC
ACERIN—RACINE	ATTICE—TIETAC	ORACLE—RECOAL/COREAL (vf)
LOCATE—ACETOL	ALCOVE—COEVAL	OCTANE—COETAN
AVOCET—VOCATE	ACURSE (vf)—SACEUR	ASCUSE—CAUSES/SAUCES
MONICA—ANOMIC	ATOMIC—MATICO	APONIC—PACINO
UNIVAC (computer coy)—VICUNA	CARUSO—ACORUS	DIALED—ALDIDE
DEAD 'UN—UNDADE	ADREFE—FEARED/DEAFER etc	HEADER—ADHERE
ELATED—DELATE	ARDERE—DEARER /REARED/READER	GAOLED—OLD AGE
ASLIDE—SAILED	AISLED—SLAIDE (vf)	ARRIDE—RAIDER
PEDALO—OPALED	MEDUSA—AMUSED	ATONED—DONATE
ARRODE—ROARED	DOMINA—AMIDON	STUDIA—AUDITS
GLEASE—EAGLES	GREASE—EAGRES	AGREES—SEARGE (vf)
ETHANE—HEATEN	ENAMEL—MELANE (vf)	ARLENE—LEANER

RELATE—ELATER	NEATER—ENTERA	TEASES—ASSETTE (vf)
TEASET—ESTATE	IMAFEN (Sted)—FAMINE/FEMINA	AFLORE—LOAFER
FAUSEN—UNSAFE	MIRAGE—IMAGER	GAINER—INRAGE
AIGRET—TRIAGE	GRAIWE (vf)—EARWIG	NEAPIL (vf)—ALPINE
SIXALE—ALEXIS	PIEMAN—IMPANE	TIEMAN (vf) INMATE
INANER—RANINE	IASPER (vf)—PRAISE	PIRATE—APERIT (Roget)
AWRITE—WAITER	KOREAN—AKORNE (vf)	ALLURE—LAUREL
NEURAL—ULNARE	LASQUE—EQUALS	VALUES—AVULSE
ENAMUR (vf)—MANURE	ARSEN—REASON	PEANUT—UN TAPE
PESARO—OPERAS	UPRAPE—PAUPER	UPRATE—TEAR UP
KAPOSI—OKAPIS	MINAUL—ALUMNI	RAISON—ARSINO
RATIOS—ARISTO	SALOON—ALONSO	BELIED—EDIBLE
EDUCED—DEDUCE	EDEMIC—DECIME	EDUCES—SEDUCE
OLD ICE—COILED	PIECES—ESPICE	OUCHES—CHOUSE
COINER—IN CORE/ORCINE (Pull)	EXOTIC—COXITE	UNESCO—CESOUN (vf)
EDMEDE—DEEMED	ELUDED—DELUDE	VEINED—ENDIVE
IRESED—DESIRE	EVOKE—DEVOKE	ELOPED—DELOPE
EMOTED—DEMOTÉ	DEPONE—OPENED	EPODES—DEPOSE
INDOLE—LEONID/LOINED	EDIPOL—DIPOLE	NOISED—ONSIDE
RUINED—UNRIDE/INDURE	NUDIES—ISUNDE	ITRODE—RIOTED
UPTIDE—TIED UP	DOURER (OSPD)—ORDURE	OUSTED—STOUDE (vf)
LEVENE (vf)—ELEVEN	VENEER—ENERVE	GENOME—EGOMEN
EVILER—REVILE	SETILE (vf)—ELITES (r)	IVERES (vf)—REVISE
REVOKE—EVOKER	OMELET—TELOME	PELOTE—OPELET
EMOTER—REMOTE	PERONE—OPENER	REUNES (vf)—ENSURE
ERNUTE (vf)—NEUTER	VETOER—OVERTE (vf)	FUSULE (vf)—USEFUL
ELOIGN—LINGOE	UGLIER—GUIRLE (vf)	REGION—ERINGO
GENIUS—ISUNGE	EGRIOT (vf)—GOITRE	GESTIO (Pull)—EGOIST
PITIES—EPISTI (vf)	UNLIKE—LEUKIN (Sted)	ELINOR—NEROLI
VIROLE—OLIVER	MINUET—IMUNTE	RIMOSE—ISOMER
EVOMIT—MOTIVE	INURES—RUSINE	REISUN—INSURE/URSINE
ROSSIE—OSIERS	SUISSE—ISSUES	MOOSES—OSMOSE
OSSEUM (Sted)—MOUSSE	NEURON—UNRONE	OUTSET—STOUTE (vf)
SUTURE—UTERUS	ORIGIN—NIGORI	OLINGO—LOGION
OOLITH—THOLOI		

Fecund 4-letter alphomes

Given a particular set of four different letters (perhaps stated as an alphome, ie letters in alphabetic order), $4 \times 3 \times 2$ ie 24 different potential words can be made. For this article, each such word must consist of two vowels and two consonants. For example, given the alphome AELT, there are six potential words starting with each of the four letters. Each of these 24 words has a particular vowel pattern, which means it has just four possible duals: for example, LATE has the potential duals ALET(vf), ATEL, ELAT (vf), and ET AL (all of which are actually words or phrases, and one of which, ET AL, is necessarily the reverse of the starting word).

Another (overlapping) example is TALE, which pairs with ATEL, ALET, ET AL, and its reverse ELAT. This maximum of four occurs about 225 times among all the alphomes: see later for more analysis and examples.

The most often such 4-pair groups occurred in a single alphome was 10 times, in AELT: ALTE (1400q under ciclatoun), ATLE, TALE, ATEL, ALET (vf), ET AL, ELAT (vf), LETA (vf), TELA (Pull), and LATE.

The second highest was eight times, in AETS: ASTE (OSNG), ETAS (pl), ATES, ASET, ETSA

(Egypt, CL), ESTA (pname, WNB), STAE (EDD/ Swedish town), and STEA (EDD).

A single alphome could give rise to $24 \times 4 = 96$ possible dual pairs, although this counts each pair twice. In fact, the most fecund alphome, AETS, had 32 pairs: ((r) means reverse):

AEST with STEA (EDD)	AETS with STEA (r)
ASET with TESA (r) & SETA	
ASTE (OSNG) with SAET (vf), SEAT, TAES (vf) & TEAS	
ATES with TESA & SETA (r)	EAST with STEA
EATS with STEA	ESTA with TEAS
ETAS with TESA (F&W teesa) & SETA	ETSA with TAES & TEAS
SAET with ETSA (CL) & ESTA (WNB)	SATE with ETAS (r), ATES, & ASET
SEAT with ETSA & ESTA (WNB)	
STAE (EDD) with AEST (vf), AETS (EDD), EAST, & EATS (r)	
TAES with ESTA	TASE (vf) with ETAS, ATES, & ASET

Note that in this layout, the eight quads above, not all listed as quads, are not easy to spot, because no pair is repeated. The total number of pairs, 32, is exactly half the theoretical maximum. The failure to reach perfection is partly because I found no pairs for the possible words ATSE, TSAE, TSEA, and ESAT (European Satellite Corp?). Note that this alphome also contains the greatest number of reversals in any alphome.

The largest number of reversal pairs in a single alphome is five, and this occurs twice, once as immediately above, and also: AERT (vf)—TREA (vf), ETAR (vf)—RATE, TARE—ERAT (as in QED), EART (vf)—TRAЕ (EDD), ARET—TERA.

Fecund 4-letter words

The most fecund word can have at most four partners. Consider the heterogram $V_1V_2C_1C_2$. There are two ways of replacing the vowels by consonants, and conversely, making $2^2 = 4$ partners maximum (in which the large 2 is really factorial 2). This is the reasoning behind the theoretical square numbers mentioned earlier. By definition, all such words are heterograms.

The most fecund word is a distinction shared by many having the maximum four partners: a small selection is:

ABUT with TUBA (r), BATU (Pull), BUTA (vf), & TABU
 ALES with LASE, SELA (r, vf), LESA (Italy, Times Gazeteer), & SALE
 ALSO with SAOL (Ch, Old Fr whence soil) SOAL LAOS (country, or Anc. Greek whence lay) & LOAS (vf)
 ANTE with TEAN, TA'EN, NEAT, & NAET (vf)
 APED with PEDA, DAPE (vf), PADE (vf), & DEPA (r)
 APES with SAPE (vf), SEPA (r, Ch Protestant), PASE (OSPD), & PESA
 APSE with SAEP (vf), SEAP (vf), PAES (vf), & PEAS
 ARSE with SAER (Pull), SEAR, RAES (vf or pl pname), & REAS
 EMIR with REMI (Latin, OED remicle; or Abraham Remi, Professor Royal of Eloquence, under OED eloquence), MIRE, RIME (r), & MERI (vf)
 ESIL with SELI (Pull), SILE, & LISE (r)
 ETAS (pl) with SATE (r), TESA (F&W teesa), & SETA
 ICED with DICE, DECI (r, prefix), CEDI, & CIDE (Sted)
 IDEM with DEMI, MEDI (r, vf), DIME, & MIDE
 ISLA (pname) with SIAL, LIAS, LAIS, & SAIL
 ISLE with LEIS, LIES, SILE, & SIEL (vf)
 LISE (vf) with ELIS, ILES, ISEL, & ESIL (r)
 MANE with ANEM (Med), AMEN, ENAM (r), & EMAN (Med)
 RASE with ERAS, ARES (pl or vf), ESAR (r, TEA), & ASER (vf)

RATE with ETAR (r, vf), ARET, ATER (vf), & ERAT (Latin, as in QED)
 SELI (Pull) with ELIS (Ancient Greek district, containing Olympia), ILES (r, OED ile), ISEL (Pull), & ESIL
 TALE with ATEL, ALET (vf), ET AL, & ELAT (r, vf)
 TARE with ETAR, ARET, ATER, & ERAT (r)
 TUNA with ANUT (r, see Ch drake), ATUN (Pull), UNAT, & UTAN (vf)

When two letters are the same (so clearly no heterograms), eg VVC_1C_2 , there are $2 \times 1 = 2$ partners maximum, eg:

ABED with BEDA (Pull) & BADE	AGAL & GALA
AGAR with GARA & RAGA (r)	ALAS with SALA (r) & LASA
ANAL with LANA (r) & ALAN (all OED)	ARAB & BARA (r)
BAAL & ALBA	DATA with A TAD (r) & ADAT
EDDA & DEAD	
ELSE & LEES, ELSE & SEEL and EELS & SLEE (r)	ESNE & SEEN
EVEN & NEVE (r)	EVER & REVE (r)
EWER & WERE	EYED & YEDE
IMID & MIDI	LAVA & AVAL (r)
LEEK & ELKE and ELKE & KEEL	MAIM & AMMI
MAYA & AMAY	
NANO & ONAN (r) and NANO & ANON	NONE & ONEN (in Gwent, or vf)
OBOY & BOYO	
OOHS & SHOO (r) and OHOS & SOHO (r)	OSLO & LOOS
ROAR & ORRA	
SARA (pname or Ch, Hebrew root of Israelite) with ARAS (r, Pull) & ASAR (Pull)	
SEAS & ASSE	SEER & ERSE
TOON (Pull) & ONTO	UDUK & KUDU (r)

When two pairs of letters are the same (so no heterograms), $VVCC$, there is just one possible partner, eg:

AKKA with KAAK	AMMA with MAAM
ARAR with RARA	BOOB with OBBO
DADA with ADAD (r)	NONO with ONON (r)
NANA with ANAN (r), and NAAN with ANNA	SEES with ESSE
SISI with ISIS (r, river in Oxford, Egyptian goddess)	TOOT with OTTO

For longer examples, the number of potential partners is the product of two combinations (in the mathematical sense). Indeed, in the category VVC_1C_2 above, the 2×1 is fundamentally

$$2! \div (1! 1!) \quad \times \quad 2! \div (2! 0!)$$

Note added in proof: ESAT Telecommunications Group plc exists, as does E-SAT Inc.