NUMBER TREES AND NETWORKS

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TWENTY-SEVEN has eleven letters, ELEVEN has six letters, SIX has three letters, THREE has five letters, and FIVE has four letters. No matter what number name one starts with, one always ends up at FOUR. Suppose instead one adds the number of letters in the name to the number itself to obtain a new number-name: TWENTY-SEVEN with eleven letters goes to THIRTY-EIGHT, and this sequence continues with FORTY-NINE, FIFTY-EIGHT, ... ending with one of the last eight hundred number names in the thousand vigintillions.

Combining all such sequences into a single network, one ends up with a structure looking like an upside-down tree with branch ends at 1,2,3,6,7,10,11,14,15... A tiny piece is illustrated below:

```
              THIRTEEN
              |   |  |
          EIGHT  TEN NINE
          |  |       |  |
     THREE  FOUR  SIX  FIVE
             |          |
            ONE        TWO
```

Notice how the different sequences combine as one moves up the network, much like the text-convergence property described by Martin Gardner in "Mysterious Precognitions" in the August 1998 Word Ways. For example, one can start with any number name between ONE and SIX, and inevitably ascend to 13; similarly, all number names from ONE through FIFTEEN eventually reach 45, and from ONE through FORTY-SIX, 105.

Similar sequences and a similar network can be created by subtracting the number of letters in the number name to determine a new one: TWENTY-SEVEN with eleven letters goes to SIXTEEN, and the sequence continues with NINE, FIVE and finally ONE. Again combining sequences, one ends up with a network of four independent right-side-up trees with beginnings ONE, TWO, THREE and FOUR. Three of these networks are very small. Interestingly, FOUR has no other numbers beside itself (unlike its fecundity exhibited in the standard convergence problem), TWO has only 7, 10 and 18, and THREE has the 26 numbers 6, 8, 12, 14, 15, 17, 20, 21, 23, 24, 25, 26, 28, 30, 32, 34, 35, 37, 40, 42, 44, 47, 49, 52, 57 and 67. The fourth network, starting with ONE which branches into FIVE, NINE and THIRTEEN, contains all remaining number names up to one thousand vigintillion.

These structures suggest a way to characterize individual number names by means of an associated tree consisting of roots and branches. We illustrate with an example. Take the number name THIRTY-FIVE and graft to it a branch consisting of those number names in the second network above THIRTY-FIVE, and a root consisting of those number names in the first network below THIRTY-FIVE:

```
 FORTY-NINE     FIFTY-TWO
          |          |
          FORTY  FORTY-FOUR
          |          |
          THIRTY-FIVE
          |          |
 TWENTY-FIVE  TWENTY-SIX
                    |      |     |
          EIGHTEEN TWENTY SEVENTEEN
            |                  |
           TWELVE            ELEVEN
            |
           SEVEN
```

Note that one counts down toward THIRTY-FIVE in the branch, and up toward THIRTY-FIVE in the root. Some trees (THIRTY-ONE, THIRTY-NINE, FORTY-ONE, etc.) have no branches, others (SEVEN, FOURTEEN, FIFTEEN) no roots, and a few exhibit no growth either up or down. The only such dwarfs from 1 to 100 appear to be TWENTY-EIGHT, THIRTY-TWO, FORTY-TWO, FIFTY-TWO, SEVENTY-ONE, and EIGHTY. On the other hand, many trees have branches reaching into the vigintillions!

There are several games one can play with these number sequences. One of the most interesting is to create sequences which, on the average, drift neither toward zero nor vigintillion. Specifically, if a number name is even, move to that number name which exceeds it in value by the number of letters in the name; if the number name is odd, move to that number name preceding it by the same count. It is not difficult to show that there is very little drift in this strategy; the total number of letters in the even number names between 1 and 100 is only four less than the total number of letters in the odd number names.

Because of this near balance, the networks generated by this strategy consist of tree-like pieces ending at cycles of two or more number names that endlessly repeat, analogous to the pentagon cycle discovered by Howard Bergerson and described on p 113-116 of Dmitri Borgmann's *Beyond Language* (Scribner's, 1967). Details of this can be found in "A Remarkable Revelation?" in the August 1995 Word Ways.

A simple example is the cycle (THIRTY-SIX, THIRTY) which has only one input sequence: THIRTY-EIGHT with eleven letters up to FORTY-NINE with nine letters down to FORTY with five letters up to FORTY-FIVE with nine letters down to THIRTY-SIX. In contrast, the cycle (TWENTY-TWO, THIRTY-ONE) captures a total of 58 names between FOURTEEN and ONE HUNDRED SIXTY-THREE. ONE HUNDRED TWENTY-FOUR begins the input sequence 144-163-143-123-102-115-98-109-95-85-75-64-73-61-53-43-33 to reach the cycle.

Hand calculations revealed only cycles of length two, but when the problem was presented to Mike Keith he programmed a computer to look for longer cycles, finding the remarkable 22-cycle depicted below. We note that some portals (entry points for input sequences) are unused by input sequences. The number names in the cycle are given in the vertical column (read them downward); input sequences are given horizontally.

```
        112-131
        128
        149
        130
        146-165-150-169-189
        164
        183
        162-181
        180
        196-176-197
    235-215-202-223-210-200-178-158-179-199
        198
    239-219-206-190
        201-182
        188-168-148
        209-229     193
156-174-195-211-192-172-191-205-186
142-160-175
        154-134
        173
    171-151-132-114
    153-133-116-137-157
```

Interestingly, the first cycle longer than 22 doesn't occur until the vicinity of 40,000, when one of length 30 is seen. This in turn is bettered by a cycle of 31 near 80,000, a cycle of 36 near 104,100, a cycle of 39 near 11,373,000, and a cycle of 40 near 40,000,000.

There are a few number names that do not converge to a cycle but fall out of the picture when their sequences reach THREE or ONE In the diagram below, all sequences converge on FIVE which then goes to ONE:

```
    6
    9   17          20
7-2-5-13-8-15-25-35-26-18-12-23-16-27
   11 10 4          37    21
   19               47
   29               57
   39               67
   28
```

Suppose that we change the rules to specify that if the number name is even one moves down in the count by the number of letters in the name, and if it is odd one moves up. As demonstrated by Mike Keith, one again ends up with a collection of cycles that absorb various number sequences, along with a few number names that do not converge to any cycle at all. Surprisingly, the only such number names of this nature are TWO which drops below zero, and ONE which rises to FOUR which goes to zero. Mike has found that there is a cycle of 8 that occurs in the 253-354 range, a cycle of 12 near 1500, a cycle of 13 near 4100, and a cycle of 38 near 101,000.

Mike notes the similarity of this up-down game to the Collatz problem in number theory. If one creates a sequence starting with any number by (1) halving it if the number is even, or (2) multiplying it by 3 and adding 1 if the number is odd, it has been conjectured that all such sequences converge to the 4-2-1 cycle. Computer studies have proven it true for all numbers less than 29200×10^{12}. Convergence to 4-2-1 seem surprising in the face of the heuristic argument which states that if x is an odd number in the sequence, the next odd number in the sequence will, on the average, have a value of x + ⅓. However, this is not an odd integer, and the probability that the next odd x will be smaller than its predecessor is in fact greater than the probability that it will be larger, tilting the sequence inexorably downward.

VERBALIA: Juegos de palabras y esfuerzos del ingenio literario

This is the title of Màrius Serra's marvelous survey of wordplay literature, published in Spanish in 2000 by Ediciones Península of Barcelona (ISBN 84-8307-321-8). (There is a proposal to publish it in English.) As the subtitle suggests, its focus is both on letter play and writing under constraint. After introductory essays on the history of wordplay and its present development in various countries, the book inventories wordplay types in the format Definition-Origin-History-Evaluation-Spanish examples:

Combinational Artifices: anagrams, reversals, palindromes, spoonerisms, multiple poems, acrostics, labyrinths and multiple acrostics, word squares, crosswords

Additive Artifices: acronyms, rhopalics, long words, slang additions, word deletions, portmanteau words, false derivations, macaronic poetry, word chains, isograms, AEIOU words, pangrams

Subtractive Artifices: transdeletions, letter deletions, text pruning, single-letter lipograms, univocalics, multiple-letter lipograms

Multiplicative Artifices: heads 'n' tails words, repeated bigrams, texts in which all words start with the same letter, echo verse, tongue twisters, isomorphs (words having the same letter pattern), alternating vowels and consonants, polygrams (no single-occurrence letters), many consecutive consonants in words, monosyllabic texts, double entendres, homophones (anguish languish), puns, linguistic boners, bilingual words (as franglais), oxymorons

Substitutive Artifices: letter-substitutions in words, word ladders, S+7 (OuLiPo word replacement in texts), cryptography, rebuses, charades, gematria (relations between numbers and letters)

As can be seen from this summary, the book combines Bombaugh's *Oddities and Curiosities of Words and Literature* with Borgmann's *Language on Vacation.* I know of no more comprehensive and up-to-date discussion on the state of European wordplay. However, he has not ignored American developments; there are more than a dozen references to Word Ways articles and monographs in the bibliography. Of Word Ways he writes "[It] is a fundamental source of Verbalia because it explores new territory from a scientific viewpoint, being written at the university level but in an appealing way. Its great virtue is to have gotten professors and word specialists in diverse areas to thoroughly examine the field of logology. Despite their advanced level, most articles have been published nowhere else— and never would have had Word Ways not existed. The thousands of pages that have been published over three decades form a remarkable corpus. One has the impression of being in virgin territory, in a new environment of human knowledge."