Ross Eckler explored planet packing in the May 2001 issue, attempting to generate an algorithm to create a string of letters from which the names of the planets could be read sequentially. Ross suggested readers might like to explore possible strings for other groups of names.

I began by tackling the names of the days of the week. The first observation is that DAY occurs at the end of each of the day names, so should go at the very end of the string we’re trying to generate. A number of letters occur only once: MOWDHFI. These can be inserted into our string at the end of the generation process. It’s worth noting that some letter sequences occur in more than one day name. TUS occurs in Tuesday and Thursday, TUR occurs in Thursday and Saturday, SU occurs in Sunday and Saturday, and TUY occurs in Tuesday, Thursday and Saturday.

Let’s start with Tuesday and Thursday. Our initial attempt to build a string looks like:

\[ \text{T H U (E, R) S D A Y} \]

Adding in Saturday merely requires the addition of SA at the front of the string, thus:

\[ \text{SA T H U (E, R) S D A Y} \]

Adding in Sunday merely requires an N adding somewhere between the U and D, so:

\[ \text{SA T H U (E, R, N) S D A Y} \]

We could have inserted the N between the second S and the D, but in anticipation of Wednesday, we put the N ahead of the second S. We can now include Wednesday by extending the string to:

\[ \text{(WED, SATHU) (R, NE) S D A Y} \]

Notice how we have now placed the E after N to accommodate Wednesday. Monday is now tackled simply by adding MO ahead of the N:

\[ \text{(MO, WED, SATHU) (FRI, NE) S D A Y} \]

Similarly, Friday is included by adding F and I around the R:

\[ \text{(MO, WED, SATHU) (FRI, NE) S D A Y} \]

That’s a total of 19 letters. The seven day names utilise a total of 15 different letters (ADEFHIMNORSTUWY). This is the absolute minimum length our target string could have. However, Wednesday has 2 D’s and 2 E’s, so the target string would need one extra D and one extra E, taking us to a string length of at least 17 letters. Saturday has 2 A’s, so the target string would need one extra A, taking us to a string length of at least 18 letters. The 19th letter, an extra S, is due to the fact that Saturday has an S before the TUR sequence, while Thursday has its S after the TUR sequence. This necessitates the sequence STURS. This explains why an S appears twice in the target string, once ahead of TUR and once after TUR. This reasoning would appear to be consistent with our actual generated string length of 19 letters.

Turn now to the colors of the rainbow. At first sight, this should be as simple as the days of the week problem—only seven names to grapple with, but actually it turned out to be rather more complicated. Let’s start with the names of the seven colors of the rainbow: red, orange, yellow, green, blue, indigo, violet.
What letter sequences occur in more than one color name? There are six:

<table>
<thead>
<tr>
<th>GE orange green</th>
<th>IO indigo violet</th>
<th>LE blue violet</th>
</tr>
</thead>
<tbody>
<tr>
<td>OE orange violet</td>
<td>RE red orange green</td>
<td>RN orange green</td>
</tr>
</tbody>
</table>

Note that some pairs of color names contain opposite sequences of letters, thus:

<table>
<thead>
<tr>
<th>EL yellow LE blue violet</th>
<th>GO indigo OG orange</th>
</tr>
</thead>
<tbody>
<tr>
<td>GR green RG orange</td>
<td>IN indigo NI indigo</td>
</tr>
<tr>
<td>LO yellow OL violet</td>
<td>NO indigo ON orange</td>
</tr>
</tbody>
</table>

With such a welter of 2-letter sequences and opposite 2-letter sequence, it's difficult to know where to start.

I decided that red, orange and green presented a good cluster of letters to start with. I may be wrong—who knows? Anyway, if we start to build the string here, what do we get? ORANGED takes care of orange and red. To incorporate green, should we build round the existing RN, or round the RE, or round the GE? This gives us three possibilities:

1. \((G, O) R (A, EE) NGED\)
2. \((G, O) RANGEE (D, N)\)
3. \(ORANGR (EEN, D)\)

Which string to choose? Each string has 10 letters, so we can't choose the shortest one. Since we now have to incorporate yellow, blue, indigo and violet, is there an obvious best color and best string to choose next?

Pick indigo. Using just the first string, we can now generate

\[
\text{INDI GO R (A, EE) NGED}
\]
\[
(G, O) R (A, EE) INDIG (ED, O)
\]
\[
(G, O) R (A, EE) INGEDIGO
\]

All have 14 letters.

Now if we choose to add violet, the first of these 3 is more attractive, since it already has IOE occurring in that order. The second and third strings can only offer IE, IO and OE. The first of the three strings can now be expanded to

\[
V \text{ INDI GO R (A, EE) L NGED T}
\]

Now add yellow, by expanding this string to

\[
(\text{VINDIGOR, Y}) (A, EE) L (LOW, NGEDT)
\]

Now add blue by inserting a B and U, thus:

\[
(\text{VINDIGOR, Y}) (A, EE) BL (LOW, UNGEDT)
\]

That's 23 letters. It's not at all clear to me whether this is optimal or not. Would I have done better by tackling the colors in a different order? Would I have done better by utilising different sets of letters that had already been built into a string, thereby adding new letters to a different part of the string?

If a particular string is optimal for the seven colors of the rainbow, what are the implications if I wanted to add an eighth color, say turquoise? At what point in the string generation process should I have done something different to accommodate turquoise?