LATTICE PLAY

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At first glance, lattices seem to offer few attractions as a wordplay form. They are far more abundant than word squares and easier to construct. Although long neglected, lattices yield a previously unknown but elegant pangrammatic set, one involving only 33 total letters and words from a single dictionary.

This new pangrammatic set becomes a second answer to a challenge issued nearly 25 years ago in the pages of Word Ways. In the February 1979 article "Word Ways Challenges," Ross Eckler and Philip Cohen asked "Can any form (diamond, pyramid) contain all 26 letters?" In February 1984,, Jeff Grant provided one answer: a pangrammatic diamond whose major axes each had length 9, containing a total of 41 letters.

Lattices actually pose several nontrivial questions. Questions about lattices can be studied using methods used in traditional word square searches, by applying a simple transformation described below. Thus, nearly any question that can be raised regarding word squares can also be asked about lattices. Such questions include: (a) are there pangrammatic lattices? (b) what is the largest lattice? (c) what are the minimum letter sets needed for various lattice lengths? I will answer the first question, suggest a lower bound answer for the second, and merely pose the third.

By a lattice I mean, of course, a collection of words all of equal length, arranged in rows and columns, in such a way that letters of each word strictly alternate between those occurring in that word along and those forming a junction with a second word. Below are shown two lattices of length seen. I call the first type open because its outside letters are contained in one word only. By contrast, the second lattice below is closed. A closed lattice always has more total letters than an open lattice of the same length. A lattice of even length is necessarily half-open.

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  V   F   P           O B J E C T S
J A C O B I N         U   O   A   I
  Q   X   C           T R I P L E X
M U Z H I K S         G   N   Q   T
  E   U   L           R E T O U C H
W R O N G E D         E   E   E   L
  O   T   S           W O R K D A Y
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The two lattices above contain only words sanctioned by the 3rd edition of the Official Scrabble Players Dictionary. Notice that the first lattice shown above is nearly pangrammatic. It lacks only the letter Y to exhaust the alphabet, containing 25 different letters among 33 total letters. The second lattice contains more letters than the first, 40 to 33, but fewer different letters, 22. Each of these lattices gives promise that a single dictionary, one moderately larger than the OSPD3, may yield a pangrammatic 7-lattice.

Webster's New International, Second Edition, is a natural candidate for the larger dictionary search. It contains some 37,000 lower-case 7-letter words, compared with 23,000 such words in the OSPD3. A search in Webster's Second yielded the following pangrammatic set.

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  B   L   J               Q   T   V
C A Z I Q U E           J U D O G I
  G   F   K               E   W   X
S W A T H E D           F A M B L E
  Y   M   B               Z   A   N
E N V A P O R           C Y P R E S
  S   N   X
```

All the words in this pangrammatic set, except BAGWYNS and ENVAPOR, are also found in OSPD3, and in the Third Edition as well as the Second of Webster's New International. BAGWYNS is a heraldry term for fantastic beasts that have the body of an antelope, the horns of a goat, and a horse's tail. ENVAPOR is the only word found exclusively in the Second Edition. JUKEBOX was a recent word for the Second Edition, found in the Addenda.

A minor side question naturally ensues. The smallest lattice with more than 26 letters is the half-open 6-lattice, which has only 27 letters. How many different letters can occur in a 6-lattice? The one above misses only the common letters H and K! CYPRES, FAMBLE and VIXENS are found in the Second Edition, QUEAZY, CYPRES and VIXENS are found in OSPD3, and TOWBAR and JUDOGI (judo garb) are found in the on-line Penguin Rhyming Dictionary, and in more than 5,000 Internet references each. One suspects that dictionary resources of the order used in Jeff Grant's "Pangrammatic Diamond" article in February 1984 would produce a pangrammatic 6-lattice.

One might think that lattices of size 9 would offer fertile ground for pangrammatic solutions. The open 9-lattice has 56 letters, and the closed 9-lattice has 65, compared to the 33 letters of an open 7-lattice. But pangrammatic 9-lattices are uncommon. The problem is that the open 9-lattice is restricted by 16 intersection points, and the closed 9-lattice is restricted by 25 intersection points. In comparison, the open 7-lattice is restricted by only 9 intersection points. Using a large dictionary such as the Second Edition produced scores of near-pangrammatic sets. Curiously, these included nearly 20 lattices whose top row was FACETIOUS, and which each contained 25 of the 26 letters of the alphabet. Only one true pangrammatic set was found in the Second Edition:

```
  C   J   F   K
G A Z E H O U N D
  V   L   R   I
S E Q U A C I T Y
  R   T   I   W
I N C O M P L E X
  O   N   A   A
B U D G E T A R Y
  S   S   E   S
```

The question of the largest lattice is really threefold: what are the largest open, closed, and half-open lattices, respectively? The search for large lattices can easily be transformed into ordinary word square searches. For instance, we can create a dictionary of 7-letter "words" by replacing each 15-letter word with its even-numbered letters, so that SUPERNATURALISM becomes UENTRLS. Then any 7-square formed by the resulting 7-letter "words" can be transformed into an open 15-lattice.

By considering this transformation, we can calculate what length lattices are reasonable. An open 15-lattice transforms into a "word square" of size 7. A closed 15-lattice, or a 16-lattice, or an open 17-lattice, all yield "word squares" of size 8. A closed 17-lattice yields a "word square" of size 9, and so on. Ross Eckler in May 1992 and Chris Long in February 1993 gave estimates of the word list size likely to yield a word square of length N. Eckler estimated the support for a 7-square to be some 5,459 words but that an 8-square would require a 15,285 word support. The supports for 15- and 16-lattices are not necessarily numerically identical to the supports of 7- and 8-squares, but one would expect the 16-lattice to require a basis of candidate words at least 3 times larger than an open 15-lattice. The Second Edition has some 7,100 candidate words for open 15-lattices, but fewer than 4,100 candidate words for 16-lattices or open 17-lattices. So Webster-restricted searches for 16-lattices or larger are unlikely to succeed.

Experience mirrors theory. Using the above word square transformation, one finds scores of open lattices of size 15, but no 16-lattices. The 15-lattices include the following lattice with a word study theme. Shown below, to the right, is the same lattice's transformation into a word square of size 7. The example has identical rows and columns, although this is not a requirement of lattices.

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  I   S   E   L   M   I   I
I N C O M P R E H E N D I N G
  C   P   I   X   T   E   E
S O P H I S T I C A L N E S S
  M   I   T   C   M   T   C            N O P E E D N
E P I S T E M O L O G I C A L          O H S I A N S
  R   T   M   G   R   F   P            P E E O O I A
L E X I C O G R A P H I C A L          E I O R P I A
  H   C   L   A   H   C   B            E A O P O A L
M E T A M O R P H O S A B L E          D N I I A I N
  N   L   G   H   S   T   E            N S A A L N S
I D E N T I F I C A T I O N S
  I   E   C   C   B   O   E
I N E S C A P A B L E N E S S
  G   S   L   L   E   S   S
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The preceding should be considered a preliminary progress report, not a definitive treatise. Pangrammatic lattices have been discovered. A starting point for the largest lattice (at least 15 letters) is reported. But many questions remain. For instance, is there a pangrammatic 6-lattice? Are there closed pangrammatic lattices? Are there lattices of size greater than 15? What is the minimum letter set for lattices of size 7 or greater?