# THE ALPHABET CUBE AND BEYOND

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Two-letter words can be plotted as points on a two-dimensional plane. The first letter of a two-letter word is plotted against the x-axis, and the second letter is plotted against the y-axis. Along the x-axis, x=1 is equivalent to the letter A, x=2 is equivalent to the letter B, and so on, with x=26 being equivalent to the letter Z. Similarly, plot the second letter of a word on the y-axis, with y=1 equivalent to A, y=2 equivalent to B, etc. So, for example, take the two-letter word HE. The H corresponds to x=8 and the E corresponds to y=5. The point (x=8, y=5), or, more compactly, (8,5), can be used to represent the word HE in two-dimensional space.

Three-letter words can be plotted in a three-dimensional space. This is essentially the subject explored in "The Alphabet Cube" introduced by Dave Morice in the May 1990 Word Ways. This time, imagine an additional axis, the z-axis. The first letter of a three-letter word can be represented by a point measured along the x-axis, the second letter by a point measured along the y-axis, and the third letter by a point measured along the z-axis. So, for example, the point (x=8, y=5, z=13), or (8,5,13), can be used to represent the word HEM.

So, by extension, four-letter words can be plotted in a four-dimensional space, a hypercube. For example, the word HEMP is represented by the point (8,5,13,16). Five-letter words can be plotted in a five-dimensional space, a hyperhypercube. For example, HEMPY is represented by the point (8,5,13,16,25). Indeed, any n-letter word can be plotted in n-dimensional space. Let's take a look at some specifics.

**Two-Letter Words**

All two-letter words can be plotted against a set of x- and y-axes. It's useful to consider the concept of how close together or how far apart two words are. The distance between two words (or points) is simply calculated using Pythagoras's theorem.

Consider two two-letter words, ab and fg. Their squared distance of separation is given by $D^2 = (f-a)^2 + (g-b)^2$. The theoretical minimum distance apart is when one of the bracketed terms is one and the other is zero. There are many examples where two two-letter words are just one unit apart: AM/AN, GO/HO, MO/NO, US/UT, XU/YU.

The theoretical maximum distance apart is when each of the bracketed terms is 25, equivalent to the separation between A and Z. Pythagoras's theorem yields the square root of $25^2 + 25^2 = 1250$, or 35.36. How close to the theoretical maximum can real-world two-letter words be? The words AA and ZO are 28.65 units apart. AA and YU are slightly farther apart at 31.24 units. However, AY and YA seem to be the farthest at 33.34 units. All these words are in Official Scrabble Words International (OSWI). By venturing into the Oxford English Dictionary, it's possible to identify a pair of words even farther apart; AA and ZY (an old form of the verb "see") are 34.66 units apart.

**Three-Letter Words**

The separation of two three-letter words in three-dimensional space can also be investigated. (If you wish, dig out Dave Morice's "The Alphabet Cube".) Consider two three-letter words, abc and fgh. Their squared distance of separation is given by $D^2 = (f-a)^2 + (g-b)^2 + (h-c)^2$. The theoretical minimum distance apart is when one of the bracketed terms is one, and the other bracketed terms are zero. There are many examples where two three-letter words are just one unit apart: CAB/DAB, ARK/ASK, RAM/RAN. Two letters remain the same and in the same positions, while a third letter is changed to an alphabetical neighbor. These are called alphagrams, betagrams and gammagrams where, respectively, the first, second, and third letters change.

The theoretical maximum distance apart is when each of the bracketed terms is 25, equivalent to the separation between A and Z. Pythagoras's theorem yields the square root of 1875, or 43.30. How close to the theoretical maximum can real-world three-letter words be? The common words ADD and ZOO are 19.44 units apart. The still-common AYE and YEW are 36.06 units apart. The less-common ABA and ZUZ are 40.14 units apart. These six words all use OSWI as their source. Again, by venturing into the Oxford English Dictionary, it's possible to identify a pair of words even farther apart; ABA and YZY (another old form of the verb "see") are 41.57 units apart.

**Longer Words**

Words of four letters and more can't be plotted on Dave Morice's introductory Alphabet Cube, the Mark 1 model, as it only worked with three-letter words. For studies in four and more dimensions, acquisition of the advanced Alphabet Hypercube Mark 2 is recommended. Dave had originally anticipated a series of different models, one for each different number of dimensions. Dave's article referred to "an 8-dimensional model." Significant technical advances during the intervening years now allow all dimensions to be handled on the one advanced model! One of the buttons on the Alphabet Hypercube is marked "number of dimensions." Keying in 4 or 7 or 45 allows the user to manipulate words in the specified number of dimensions. Keying in 3 merely allows the Alphabet Hypercube to default to the basic mode of operation on the original three-dimensional Alphabet Cube.

As with two- and three-letter words, the closest that two n-letter words can be is just one unit apart. These are alphagrams, betagrams, gammagrams, and so on, where the letter change involves two neighboring letters of the alphabet. Examples include words ending in ED/EE, ER/ES, and IC/ID. Examples of words one unit apart are given in the table on the next page for all word lengths.

How about examples of pairs of words that are far apart? The trick is to identify pairs of words that have complementary arrangements of letters from early and late in the alphabet. For every instance of the letter A in a word, can another word be found with a Z in the same position? Or failing that, a Y or an X? And if not an A, how about a B?

From the table, it can be seen that the four-letter words ABBA and ZIZZ are 43.30 units apart, against a theoretical maximum of 50. This pair has two A-Z pairs, a B-Z pair, and a fairly weak B-I pair. Is there another pair of four-letter words farther apart than this pair?

From the table, BUZZY and YACCA are 50.63 units apart, against a theoretical maximum of 55.90 units. It is interesting to note that BUZZY and YACCA are closer to the five-letter theoreticcal maximum than ABBA and ZIZZ are to the four-letter maximum.

As the word length increases, the gap between the theoretical maximum and the real-world maximum should increase. With increasing length, it is less likely that two words exist with complementary arrangements of letters. At the six-letter level, the CABALA and SYZYGY pair suffers from the relatively small gap between L and G.

Indeed, notice how the absolute value of the distance peaks at 60.28 units for CABBAGY and ZYZZYVA, and then starts to decrease to the low to mid-50s for longer words. As word lengths increase—say, 20 letters or more—I conjecture that it is possible to find pairs of words that are farther apart than 60.28. Anyone care to suggest the greatest separation for two words?

| n | Theoretical minimum separation | Theoretical maximum separation = 25 root n | Minimum separation found | Example of minimum separation | Maximum separation found | Example of maximum separation |
|---|---|---|---|---|---|---|
| 2 | 1 | 35.36 | 1 | mo / no | 33.94 | ay / ya |
| 3 | 1 | 43.30 | 1 | cab / dab, ark / ask, ram / ran | 40.14 | aba / zuz |
| 4 | 1 | 50.00 | 1 | late / mate, alps / amps, lame / lane, pass / past | 43.30 | abba / zizz |
| 5 | 1 | 55.90 | 1 | shine / thine | 50.63 | buzzy / yacca |
| 6 | 1 | 61.24 | 1 | fallow / gallow | 50.84 | cabala / syzygy |
| 7 | 1 | 66.14 | 1 | alights / blights | 60.28 | cabbagy / zyzzyva |
| 8 | 1 | 70.71 | 1 | parsable / passable | 58.39 | abdicate / zyzzyvas |
| 9 | 1 | 75.00 | 1 | addressed / addressee | 51.71 | czaritzas / zamindari |
| 10 | 1 | 79.06 | 1 | constrains / constraint | 51.18 | ablatively / zygodactyl |
| 11 | 1 | 82.92 | 1 | compromiser / compromises | 50.00 | axiomatizes / sabbatizing |
| 12 | 1 | 86.60 | 1 | swashbuckler / swashbuckles | 50.44 | analyzations / zooxanthella |
| 13 | 1 | 90.14 | 1 | intelligencer / intelligences | 53.56 | analyzability / zooxanthellae |
| 14 | 1 | 93.54 | 1 | counterpuncher / counterpunches | 55.00 | arabicizations / westernization |
| 15 | 1 | 96.82 | 1 | photosensitizer / photosensistizes | 51.22 | acclimatization / volatilizations |
| 16 | 1 | 100.00 | 1 | intellectualizer / intellectualizes | 53.39 | acclimatizations / universalization |

**REVERSALS**

What reversal pairs are the closest and farthest apart? At the two-letter level, the closest pairs are AB/BA, DE/ED, NO/ON, OP/PO, all 1.41 units apart. The most widely separated pair is AY/YA, 33.94 units apart.

At the three-letter level, the middle letter is inconsequential. Consider the pair of words abc and cba. The squared distance between these is $D^2 = (c-a)^2 + (b-b)^2 + (a-c)^2$ . Since b-b is zero, this reduces to D = 1.41(c-a). Therefore, the closest that any three-letter reversal pair can be will be where the first and last letters are adjacent in the alphabet: ARB/BRA, BAC/CAB, COD/DOC, LAM/MAL, MAN/NAM, MON/NOM, NOO/OON, RAS/SAR, RES/SER, SAT/TAS, SET/TES, SIT/TIS. The distance apart in these examples is 1.41 units, the theoretical minimum distance for a three-letter reversal pair. The farthest apart that a reversal pair can be will occur where the first and third letters are farthest apart in the alphabet. BOY/YOB is 32.63 units apart; the theoretical maximum separation is 35.36, achieved by AnZ/ZnA, where n can be any letter.

The case of four-letter reversals is more complicated than that of three-letter ones, as there is no automatic canceling of a middle letter. The squared distance between abcd and dcba is given by $D^2 = 2(d-a)^2 + 2(c-b)^2$. Take the reversal pair SPAT/TAPS. S and T are one unit apart and P and A are 15 units apart, so the distance is equal to the square root of (2 + 450), or 21.26. The closest that a pair of four-letter reversals can be is when the second and third letters are the same and the first and fourth letters are as near neighbors in the alphabet as possible: REES/SEER, 1.41 units apart. Alternatively, the first and fourth letters can be the same and the second and third letters alphabetic neighbors: ALMA/AMLA. The farthest apart that a pair of reversals can be is when the distances between the first and fourth letters, and also between the second and third letters, are maximized. The theoretical maximum is 50 units, achieved by AAZZ/ZZAA; the most widely separated real-world examples include SWAY/YAWS at 32.25 units and DREY/YERD at 34.93.

Five-letter reversals are similar to three-letter ones—the middle letter doesn't count. If the words are abcde and edcba, then $D^2 = 2(e-a)^2 +2(d-b)^2$. In PARTS and STRAP, S and P are 3 units apart and A and T are 19 units apart; therefore, the distance is the square root of $2(3)^2 + 2(19)^2$, or 27.20. The closest that a pair of five-letter reversals can be is when the second and fourth letters are the same, and the first and fifth letters are as near neighbors in the alphabet as possible, for example REXES/SEXER which is 1.41 units apart. Or, the first and fifth letters can be the same, and the second and fourth letters as near neighbors as possible. The farthest apart that a pair of reversals can be is when the distances between the first and fifth letters and between the second and fourth letters are maximized. The theoretical maximum is achieved by the imaginary words AAnZZ/ZZnAA with a separation of 50 units, the same as for four-letter reversals. Real-world examples that are widely separated include DARTS/STRAD at 34.23 units and AYAHS/SHAYA at 35.01. Any improvements possible?

The calculation for the separation between any six-letter reversal pair is similar to that for four-letter ones. The theoretical minimum separation is 1.41, achieved by SENNET/TENNES. The theoretical maximum separation would be achieved by the imaginary pair AAAZZZ/ZZZAAA at 61.24 units. The farthest apart real-word pair found so far is DEPART/TRAPED at 36.06 units.

The calculation for the separation between any seven-letter reversal pair is similar to that for five-letter ones. The theoretical minimum is 1.41, but the closest real-world pair found so far is SAGENES/SENEGAS, at 11.40 units. It should be possible to reduce this. The theoretical maximum separation would be achieved by the imaginary pair AAAnZZZ/ZZZnAAA at 61.24 units. The farthest apart real-world pair found so far is DESSERT/TRESSED at 35.24 units.

**TRANSPOSALS**

For words from three to sixteen letters, what pairs of transposals are closest together and farthest apart? The table below summarises the closest and farthest transposal pairs we have been able to find. (Two-letter transposals are the same as two-letter reversals.) For all lengths up to and including 13 letters, transposals that are the theoretical minimum apart, 1.41 units, can be found. The examples range from SIT/TIS to MONOGRAPHICAL/NOMOGRAPHICAL. All examples involve the interchange of two letters which are alphabetically adjacent. However, for lengths 14 and 16, we haven't managed to find transposal pairs that achieve the theoretical minimum. Are there improvements on the offerings here?

| n | Theoretical minimum separation | Theoretical maximum separation: 25 root n (n even); and 25 root (n-1) (n odd) | Minimum separation found | Example of minimum separation | Maximum separation found | Example of maximum separation |
|---|---|---|---|---|---|---|
| 2 | 1.41 | 35.36 | 1.41 | no / on | 33.94 | ay / ya |
| 3 | 1.41 | 35.36 | 1.41 | sit / tis | 33.94 | ray / rya |
| 4 | 1.41 | 50.00 | 1.41 | rise / sire | 43.29 | gazy / zyga |
| 5 | 1.41 | 50.00 | 1.41 | tares / taser | 41.59 | bazar / zabra |
| 6 | 1.41 | 61.24 | 1.41 | chares / chaser | 41.59 | bazars / zabras |
| 7 | 1.41 | 61.24 | 1.41 | recures / securer | 42.05 | saxtuba / subtaxa |
| 8 | 1.41 | 70.71 | 1.41 | undefide / undefied | 41.83 | bizarres / braziers |
| 9 | 1.41 | 70.71 | 1.41 | electrode / electroed | 46.54 | broadway / wayboard |
| 10 | 1.41 | 79.06 | 1.41 | recusances / securances | 46.54 | broadways / wayboards |
| 11 | 1.41 | 79.06 | 1.41 | monocracies / nomocracies | 42.83 | broadcaster / rebroadcast |
| 12 | 1.41 | 86.60 | 1.41 | deifications / edifications | 42.83 | broadcasters / rebroadcasts |
| 13 | 1.41 | 86.60 | 1.41 | monographical / nomographical | 41.33 | exploitations / sexploitation |
| 14 | 1.41 | 93.54 | 4.24 | conservational / conversational | 40.35 | interrogatives / tergiversation |
| 15 | 1.41 | 93.54 | 1.41 | monographically / nomographically | 37.42 | autoradiography / radioautography |
| 16 | 1.41 | 100.00 | 12.57 | impressibilities / permissibilities | 37.42 | autoradiographic / radioautographic |

The table shows examples of transposal pairs where the distance between the individual words is as large as possible. For example the pair BAZAR/ZABRA is 41.59 units apart, compared with a theoretical maximum of 50. The farthest distance apart appears to peak at 46.54 units for BROADWAY/WAYBOARD and then starts to fall off, just as the number of transposals for longer words starts to fall off, reducing the number of available transposal pairs. Can any of the transposal pairs be bettered? It seems quite likely! Is 46.54 the maximum achievable in practice?

**Multiple Transposals**

What about words that have multiple transposals? For example, there are eleven transposals of the letters AEINRST in OSWI. Which pair of transposals is closest together? And which pair is farthest apart? From the table below, the closest pair is RATINES/RETINAS at 5.66 units, and the farthest pair is ANESTRI/RESIANT, at 34.12. The first pair can be deduced by inspection: both words share the pattern R-TIN-S, so their distance apart is simply calculated as the square root of $2(A-E)^2$, or 5.66.

| | anestri | antsier | nastier | ratines | resiant | retains | retinas | retsina | stainer | starnie | stearin |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **anestri** | - | - | - | - | - | - | - | - | - | - | - |
| **antsier** | 24.41 | - | - | - | - | - | - | - | - | - | - |
| **nastier** | 30.10 | 18.44 | - | - | - | - | - | - | - | - | - |
| **ratines** | 32.98 | 24.17 | 12.81 | - | - | - | - | - | - | - | - |
| **resiant** | **34.12** | 24.90 | 17.38 | 16.37 | - | - | - | - | - | - | - |
| **retains** | 34.00 | 27.86 | 21.82 | 13.64 | 11.40 | - | - | - | - | - | - |
| **retinas** | 33.47 | 22.63 | 14.00 | **5.66** | 18.44 | 16.06 | - | - | - | - | - |
| **retsina** | 28.21 | 27.20 | 20.10 | 23.37 | 22.93 | 25.46 | 24.86 | - | - | - | - |
| **stainer** | 27.60 | 29.09 | 29.26 | 26.91 | 28.35 | 27.53 | 24.58 | 32.89 | - | - | - |
| **starnie** | 22.58 | 30.53 | 30.40 | 31.87 | 32.40 | 33.50 | 30.46 | 25.57 | 16.31 | - | - |
| **stearin** | 28.18 | 31.97 | 32.50 | 26.61 | 28.91 | 24.12 | 24.90 | 32.40 | 11.31 | 20.05 | - |
| | **anestri** | **antsier** | **nastier** | **ratines** | **resiant** | **retains** | **retinas** | **retsina** | **stainer** | **starnie** | **stearin** |

There are ten AEGINRST transposal pairs in OSWI. Which is closest together, and which farthest apart? Examination of the table below reveals that the closest is GRANITES/INGRATES at 11.66 units, and the farthest apart is GRANITES/TASERING at 35.92.

| | angriest | astringe | ganister | gantries | granites | ingrates | rangiest | reasting | stearing | tasering |
|---|---|---|---|---|---|---|---|---|---|---|
| **angriest** | - | - | - | - | - | - | - | - | - | - |
| **astringe** | 25.38 | - | - | - | - | - | - | - | - | - |
| **ganister** | 29.33 | 28.04 | - | - | - | - | - | - | - | - |
| **gantries** | 23.49 | 26.57 | 15.62 | - | - | - | - | - | - | - |
| **granites** | 22.93 | 25.50 | 24.17 | 26.38 | - | - | - | - | - | - |
| **ingrates** | 23.45 | 23.62 | 25.06 | 25.22 | **11.66** | - | - | - | - | - |
| **rangiest** | 25.06 | 34.93 | 25.50 | 24.17 | 32.40 | 30.10 | - | - | - | - |
| **reasting** | 27.17 | 32.34 | 27.02 | 23.15 | 27.96 | 30.10 | 25.69 | - | - | - |
| **stearing** | 30.72 | 31.59 | 31.21 | 34.23 | 27.57 | 32.62 | 27.75 | 23.87 | - | - |
| **tasering** | 33.67 | 31.84 | 23.11 | 25.38 | **35.92** | 35.19 | 18.00 | 23.32 | 23.96 | - |
| | **angriest** | **astringe** | **ganister** | **gantries** | **granites** | **ingrates** | **rangiest** | **reasting** | **stearing** | **tasering** |

## ALPHOMES

Readers will be familiar with alphomes, also known as reduced letter forms, where the letters of a word are ordered in alphabetic sequence. For example, AEINRST is the alphome of eleven words in OSWI, and numerous others when additional reference sources are added.

Suppose the alphomes of all n-letter words are plotted in n-dimensional space. For any given value of n, how far apart are any pair of alphomes? Which pair is the closest? And which pair the farthest apart? Finding pairs of alphomes only one unit apart is easy. There are thousands of substitute-letter transposals involving neighboring alphabetic letters that don't change the order of the alphomes involved. At the seven-letter level, the alphome AEINRST is only one unit away from ADINRST (from INDARTS), AEHNRST (from ANTHERS and others), AEIMRST (from MAESTRI and other), and so on.

Finding pairs of alphomes that have the maximum separation is almost as easy, though some trial and error is required. The publisher Chambers produces a Scrabble players guide called Official Scrabble Lists (the latest edition is the 3rd). This lists all the seven- and eight-letter alphomes for words in its sister publication Official Scrabble Words (4th edition). The first of the seven-letter alphomes is AAAADNP (from APADANA), and the last of the seven-letter alphomes is ORSTTUU (from SURTOUT). The distance between AAAADNP and ORSTTUU is 38.73 units. An alphabetically-later seven-letter alphome is AAABBCL (from CABBALA); the distance between this and ORSTTUU is even greater, 43.15 units. Now substitute ORSSUUU (from USUROUS) for ORSTTUU—the distance between AAABBCL and ORSSUUU is fractionally greater, 43.17 units. Is there another pair of alphomes more widely separated than these two?

What about eight-letter alphomes? Again, finding pairs separated by only one unit seems easy. There are thousands of substitute-letter transposals involving neighboring letters, for example AEGINRST/AEHINRST. But which pair of eight-letter alphomes is the most widely separated? Here are some trial-and-error pairs:

| | |
|---|---|
| AAAACCRR (from CARACARA)/RRSSSUUU (from SUSURRUS) | 42.71 units |
| AAABBHKL (from KABBALAH)/RRSSSUUU (from SUSURRUS) | 42.78 units |
| AAABEGHL (from GALABEAH)/RRSSSUUU (from SUSURRUS) | 42.81 units |
| AAABBCLS (from CABBALAS)/MSSTTUUU (from TSUTSUMU) | 43.00 units |
| AAABBCLS (from CABBALAS)/NRSTTUUY (from UNTRUSTY) | 43.26 units |
| AAABBCLS (from CABBALAS)/RRSSSUUU (from SUSURRUS) | 43.46 units |

The reader is invited to improve on the seven- and eight-letter alphomes here, and to explore alphomes of other lengths.

## SQUARES

Let's go back to simple two-dimensional space where two-letter words can be plotted against an x-axis and a y-axis. Is it possible to find a set of four two-letter words such that the positions of the four words allow a square to be traced out?

Squares can have two possible orientations. The simpler orientation is where the sides of the square are parallel to the x- and y-axes. A number of such squares have been found. A square of side one can be traced out by the four two-letter words OH, OI, PH, PI, all found in OSWI. A larger square, with sides of four units, can be traced out by AA, AE, EA, EE, also found in OSWI. An even larger square, with sides of 14 units, can be traced out by BA, BO, PA, PO. This is the largest square with all four words in OSWI. However, a square with sides of 24 units can be traced out by BA, BY, ZA, ZY, the first two words being in OSWI and the last two in the OED.

The more complex orientation is where the sides of the square are not parallel to the x- and y-axes. As the corners of the squares can only have integral coordinates, there are only a limited number of square sizes and orientations possible, though they can be anchored at various points. These are all determined by Pythagoras's theorem $a^2 + b^2 = c^2$ where a, b and c have integral values. The only sets of a, b, c values satisfying (1) Pythagoras's theorem, (2) the need for integral values, and (3) the need for the square to fall fully within the alphabet's bounds—that is, from (1,1) to (26,26)--are (3,4,5), (4,3,5), (6,8,10), (8,6,10), (9,12,15), (12,9,15), (5,12,13) and (12,5,13).

Consider a two-letter word anchored at (m,n). The coordinates of the four corners are given by (m,n), (m-a, n+b), (m-a+b,n+b+a) and (m+b,n+a). in order to find squares where all four corners correspond to real two-letter words, it is necessary to trial various values of m and n, along with the associated values of a and b listed above. Here are some near-misses where three of the four corner words are valid:

| | |
|---|---|
| DA,AE,EH,HD | anchored at DA, a=3, b=4; HD not valid |
| EA,AD,DH,HE | anchored at EA, a=4, b=3; DH not valid |
| EA,BE,FH,ID | anchored at EA, a=3, b=4; FH not valid |
| HA,BI,JO,PG | anchored at HA, a=6, b=8; PG not valid |
| KA,HE,LH,OD | anchored at KA, a=3, b=4; LH not valid |
| MA,ID,PE,LH | anchored at MA, a=4, b=3; LH not valid |

Is anyone able to find a square with sides not parallel to the axes where all four words are valid?

**CUBES**

The concept of a square in two-dimensional space where all four corners are valid words can be extended to three-dimensional space. The square now becomes a cube with eight corners, and the words at these corners are three-letter words. Are there any such cubes?

Consider cubes where the sides are parallel to the x-, y- and z-axes. Assume that one corner of the cube is at (m,n,p) and that the length of the sides of the cube is q. The eight corners are at (m,n,p), (m+q,n,p), (m,n+q,p), (m,n,p+q), (m,n+q,p+q). (m+q,n,p+q), (m+q,n+q,p), and (m+q,n+q,p+q). Can values be found for m,n,p and q which generate eight valid words? It's worth noting that A and E are four units apart, as are E and I. Furthermore, I is six units from O.

If we set n=A, can we find suitable values for m and p? Setting m=B and p=N, we generate the words BAN,BAR,BEN,BER,FAN,FAR,FEN,FER. All eight words are in Webster's Third, and all but BER are in OSWI. Setting m=P and p=N, we generate the words PAN,PAR,PEN,PER, TAN,TAR,TEN,TER. All are in Webster's Second and the OED, and all but TER are in OSWI.

If we set n=E, can be find suitable values? Setting M=H and p=P, we generate the words HEP, HET,HIP,HIT,LEP,LET,LIP,LIT. All eight are in OSWI, and all but LEP are common words.

If we set n=I, can we find suitable values for m and p, bearing in mind that I and O are six units apart? Setting m=M and p=L, we generate MIL,MIR,MOL,MOR,SIL,SIR,SOL,SOR. Six of these are in OSWI, SIL is in Webster's Third and SOR is in the OED. Setting m=M and p=N, we generate MIN,MIT,MON,MOT,SIN,SIT,SON,SOT. All are in Webster's Third, and all but MIN and MIT are in OSWI.

What about cubes of larger size than 4 or 6? The use of A and O requires that the cube be of size 14. If we generate BAD,BAR,BOD,BOR,PAD,PAR,POD,POR, seven words are in OSWI and POR is in Webster's Second and the OED. If we generate FAB,FAP,FOB,FOP,TAB,TAP,TOB, TOP, all words are in the OED and seven (all but FAB) are in OSWI and Webster's Third. If we generate FAD,FAR,FOD,FOR,TAD,TAR,TOD,TOR, seven words are in OSWI and FOD is in Webster's Second. All eight are in the OED, but FOD is hidden away as an obsolete variant spelling of FOOD.

Is anyone able to find a cube with sides not parallel to the x-, y- and z-axes? Or, more generally, a rectangular box having sides of different lengths?

## HYPERCUBES

Although it's impossible to visualize, we can mathematically describe a hypercube in four-dimensional space. It will have 16 corners--the number of corners doubles every time a further dimension is added. Are there any hypercubes with valid four-letter words at all 16 corners? We present two partial solutions to this problem

Consider the set of words

LEAN LEAR LEEN LEER LIAN LIAR LIEN LIER
PEAN PEAR PEEN PEER PIAN PIAR PIEN PIER

12 of which are in OSWI. LEEN is in Webster's Second and the OED, LIAN is found in an OED quote at "kus-kus", PIAR is in the Times Index-Gazetteer, and PIEN is in both Webster's Second and Third. Or, consider the set

LAEN LAER LAIN LAIR LEEN LEER LEIN LEIR
PAEN PAER PAIN PAIR PEEN PEER PEIN PEIR

10 of which are in OSWI. LAEN, LEIN, PAEN and PEIR are in the OED, LEEN is in Webster's Second and the OED, and PAER is found in OED quotes at "pig" and "pole". Which solution is better we leave the reader to judge.

Perhaps one can find a rectangular hyperbox in four dimensions with better words at its sixteen corners. However, the likelihood of finding a hyperhypercube, or even a rectangular hyper-hyperbox, having valid five-letter words at all 32 corners seems extremely unlikely.