ALPHANUMERICALLY TRUTHFUL EQUATIONS

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Susan Thorpe's pioneering excursion into alphanumeric arithmetic (WW '04-78) inspired me to look at the flip side of the coin and explore the alphanumerics of verbal equations. This involves adding up the numerical values of their letters using A=1 thru Z=26.* I began with the anagram **ELEVEN + TWO = TWELVE + ONE** reported by Melvin O. Wellman in the April '48 issue of *The Enigma*, journal of the National Puzzlers League. This is often regarded as the most perfect of all English anagrams. I reported several numero-logological coincidences regarding it in WW '02-308. I now report five more remarkable coincidences: three straightforward alphanumerics (lines 5, 15/19, 21/22 below) and two partly contrived after '02-308 (6, 8/11).

1.		**11 + 2 = 12 + 1**	
2.	E L E V E N + T W O = T W E L V E + O N E		
3.	(5+12+5+22+5+14) + (20+23+15) = (20+23+5+12+22+5) + (15+14+5)		
4.	Word for word these sum to	63 + 58 = 87 + 34	
5.	for a total of	**121 = 121,**	the digits of #1! Moreover,
6.	factoring the left side gives	$\mathbf{11^2 = 121}$	which by "logological
1.	division" recreates #1:	11+2 = 12+1.	
7	Summing the digits in #4,	6+3+5+8 = 8+7+3+4,	gives an equality
8.	whose value,	**22 = 22,**	yields another coincidence,
	if a bit "twisted". In '02-308		I showed that, if the terms
9.	of #1 are reversed to	1+12 = 2+11	and then re-reversed
10.	character by character to	11+2 = 21+1	and a 45° twist given to the
11.	first plus sign,	11**x**2 = 21+1,	
8.	the result,	22 = 22,	equals #8 above.

Equation 3 hides two more coincidences. Summing its digits

12.	(5+1+2+5+2+2+5+1+4) + (2+0+2+3+1+5) = (2+0+2+3+5+1+2+2+2+5) + (1+5+1+4+5)		
13.	word for word gives	27+13 = 24+16	
14.	which totals to	40 = 40.	
15.	Summing these digits gives	**4 = 4**	which is the sum of
16.	the digits of #5,	1+2+1 = 1+2+1	
17.	and of #8,	2+2 = 2+2	
18.	and of #21 (below),	1+3 = 1+3,	and coincidentally
19.	of the digits of #1!	[1+1+2=] **4 = 4** [=1+2+1]	
20.	Further, the digits of #13,	2+7+1+3 = 2+4+1+6,	
21.	sum to	**13 = 13**	which is the solution to
22	the original equation!	[11+2=] **13 = 13** [=12+1]	

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* In addition to alphanumerics, here's another way of turning letters into numbers:
LETTERS LITTERS LISTERS LISPERS LIMPERS LUMPERS LUMBERS **NUMBERS**

Other Registers

To explore the generality of these coincidences, I did this same set of operations with the other twenty-five numerical registers of the alphabet, using circular frame shifts. Thus register A=3, for example, means B=4, ...X=26, Y=1, Z=2.

The primary alphanumerical sums ('alphas') for line #5 ranged from 85 (ie, 85=85) to 154--with a median of 121! (Another coincidence?). Only one other register, A=26, gave a sum containing the numerals 1, 1 and 2. This reinforces the improbability of register A=1 having done so. And A=26 fails to permit the coincidence of line #6, confirming the latter's uniqueness to A=1.

Here are the (half) sums of line #5 for A=1 thru A=26 respectively. Vertical bars mark the points where a letter from the anagram passes from value 26 to value 1. Note the lack of repeats.

121, 130, 139, 148 | 131 | 114, 123 | 106, 115, 124, 133, 142 | 125 | 108, 117 | 100, 109, 118, 127, 136, 145, 154 | 85, 94, 103, 112

Between bars each step adds 9, the number of letters in the half anagram. Crossing a bar also adds 9 but subtracts 26--or 3x26 when E crosses.

Only four other registers agreed with A=1 in having the (half) value 4 in line 15 in agreement with line 19. (Line 19 of course, like lines 1 and 22, was the same in all cases, not being an alpha.) Three registers had 22 in line 8, three had 40 in line 14 and five had 13 in line 21. Only one register, A=24, agreed with A=1 in all four lines, 8, 14, 15 and 21 (but not 5).

Lines 5 and 15 are necessarily equalities since we're dealing with an anagram. However, lines 8 and 21 were inequalities in many of the twenty-six registers, with sums of 22≠13, 25≠16, 16≠7, 27≠18, 10≠19, 19≠10 and 13≠4 seen. Note that these all become equalities by adding up the digits on either side. Indeed, for each of the registers the reduced, single digit half-sum (#15) was the same for all relevant lines from #3 to #21. For successive registers it ranged from 4 to 5 to 6 to 7 to 8 to 9 to 1 and back to 4. It changed whenever one of our letters passed from 26 to 1. It skipped 2 and 3 because that jump was the triple letter E crossing the bar. The reduced sum 4 of A=1 was seen in seven other registers, reflecting the distance from E back to W in the shift cycle.

Finally, I ran the operation on the complementary or reverse register, A=26, B=25, ...Y=2, Z=1. Curiously, the results for lines #5, 8, 14 and 15 were exactly one more than the results for A=1, namely 122, 23, 41 and 5, while line #21 was an inequality, 14≠5.

The occurrence of inequalities in lines #8 (in 9/27 registers), #21 (in 6/27) and #14 (not here but in the next section) means that not only the values but the very fact of equality is a minor coincidence. Counting these three that's eight coincidences all up! The true register A=1 is clearly in all ways the most synchronistic for alphanumeric analysis of this anagram.

Other Equations

To pursue other alphanumeric equations as well as to test the anagram's coincidences further, I wished to replicate the analysis on other anagram equations. But this is the only known equation that's an anagram--excluding redundancies like ELEVEN+TWENTY-TWO = TWELVE+TWENTY-ONE and tautologies like FOURTEEN + SIX = SIXTEEN + FOUR. So instead I looked to <u>non-anagrams</u> for equations that were doubly "truthful" in that their alphas were also equal. Penultimately I was looking for triple truthfulness--a relationship between the two sums themselves, alpha and numeric, as found with the anagram.

That relationship cannot be actual equality of the two sums in the range 0-100 since alphas are all much larger than their numbers--except for 50 (alpha 66), 80 (74), 90 (87) and 100 (108, or 74 without the "one"). I couldn't find a doubly truthful equation by mixing these four but I found a truthful sum: HUNDRED + EIGHTY+ FIFTY+ FIFTY and its alpha both = 280. (One hundred purists won't like it!) On the other hand it should be possible to find a triply equal equation using larger numbers. I couldn't readily find one so I leave this as a challenge to computer savvy logologists.

There is no number that is alphanumerically truthful about itself. Closest is TWO HUNDRED FIFTY-THREE with an alpha of 254. But inserting an "and" lets 251 and 259 be truthful, as noted by Dmitri Borgmann in *Beyond Language* (Scribner's, 1967), page 114.

Failing to find a triply equal equation (n=n, a=a, n=a) , I sought a relationship in doubly truthful equations whose two sums have the same "reduced sum" like lines #15 and 19 in the anagram table. They in turn might be equals at some intermediate step of reduction, such as lines #21 and 22. A reduced sum, recall, is obtained by adding a number's digits together, repeatedly if necessary, until a single digit results. No digit reduces to itself. (ZERO tries to by reducing to 10 on the way to 1.) Only ten numbers under 100 reduce to the same digit as their alphas (14, 32, 34, 46, 52, 54, 55, 63, 78, 81). 10/100 is close to the 1/9 of random expectation.

To find a collection of three to four term (a+b=c+d) doubly truthful equations I compared every pair of integers totalling fifty or less and their sums, looking for alpha matches. For seven to eight term equations I compared every *pair of pairs* totalling eighty or less (forty each) to create hybrids thusly: if A - B = x and C - D = x then A+D=B+C, where ABC&D are pairs all with the same sum, or the single sum itself. Five, six and many seven term equations came from cancelling out redundancies in the eights, hence they are more scarce in my collection. This method isn't comprehensive anyhow and leaves heaps of five to eights yet to be uncovered, especially all of those with an odd total. But it did get all the threes and fours totalling fifty or less, and enough fives to eights for my purposes--ie, unless odd totals are somehow systematically different from evens.

I found only one doubly truthful equation of three-terms (TWENTY = FIFTEEN + FIVE) but several with four to six and over three hundred with seven to eight terms, including occasional triple equations like 19+14+5 = 13+10+9+6 = 18+15+4+1, all three with alphas of 232. (To save space I'll use numerals from here on rather than words, but you should think of the equations as spelled out.) I wasn't so thorough that I can give you a firm total, but the proportion of finds that agreed in their reduced sums seemed to be consistent with random distribution.

Doubly truthful equations are unlimited in size. For instance, smaller equals or redundancies can be added to both sides indefinitely as in this 8+8: 32+24+21+16+13+5+4+3 = 25+23+22+15+14 +12+6+1, which, after cancelling out redundancies, reduced to this 2+3: 34+13 = 20+15+12.

Here's my whole crop of three to four term doubly truthful equations and all the larger ones which after cancelling redundancies were triply truthful, ie reduced sums agreed. To find quadruply truthfuls I ran a full analysis on them as in the anagram table. For comparison I did the same on three redundant or tautological anagram equations. Column heads refer to the equivalent line #s in the anagram table. Single numbers mean the two sides of the equation were equal, two numbers mean unequal. **Bold** face marks a coincidence. (Internal agreements between columns #21, 8 and 14 involve partly reduced sums of the same initial numbers and are not of interest, ie not coincidences.)

equation (table 1 line #:)	5	8	14	15	19	21	22
reduced sums agree (15=19)							
12+1 = 11+2 (the anagram)	**121**	**(22)**	40	**4**	**4**	**13**	**13**
18+3 = **12+9**	**129**	**21**	75, 48	**3**	**3**	**21**, 12	**21**
27+3 = 30+0+0	228	21	84, 93	**3**	**3**	21, **30**	**30**
34+34 = 19+18+16+15	320	14,50	122, 167	**5**	**5**	14, 32	68
21+15+6 = 34+5+3	258	24	105, 114	**6**	**6**	24	42
16+9+7 = 11+8+8+5	203	**32**, 41	95, 113	**5**	**5**	23	**32**
14+13+7 = 15+12+4+3	268	**34**, 43	115	**7**	**7**	25, **34**	**34**
21+15+6 = 18+12+9+3	258	24, **42**	105, 123	**6**	**6**	24, 33	**42**
19+14+13 = 25+8+8+5	289	37, **46**	127, 136	**1**	**1**	19	**46**
22+15+13 = 35+11+4+0	329	41, 32	122, 140	**5**	**5**	32	50
37+15+2 = 19+18+12+5	288	36, 45	117, 135	**9**	**9**	27	54
27+18+11 = 34+10+8+4	308	29, 38	128	**2**	**2**	29, 20	56
47+13+0 = 39+12+8+1	312	42	132	**6**	**6**	33, 24	60
12+9+6+3 = 11+8+7+4	237	21,39	102, 111	**3**	**3**	**30**	**30**
18+10+8+0 = 13+9+9+5	225	45, **36**	117	**9**	**9**	27	**36**
33+11+3+1 = 28+8+6+6	309	39	138	**3**	**3**	39, 30	48
19+16+14+1 = 35+8+7+0	320	41	140, 149	**5**	**5**	23, 32	50
36+12+5+1 = 27+10+10+7	315	36, 45	126, 108	**9**	**9**	27	54
33+11+7+5 = 19+19+16+2	326	38, **56**	146, 137	**2**	**2**	38, 20	**56**
35+13+7+5 = 29+12+11+8	348	42, 51	159, 132	**6**	**6**	33, 24	60
34+19+16+1 = 25+25+10+10	376	43, 52	160, 124	**7**	**7**	25, 16	70
24+19+16+11 = 35+35+0+0	412	52, 34	160, 178	**7**	**7**	25, 34	70
35+34+4+3 = 29+19+16+12	418	31, 58	175. 157	**4**	**4**	31, 22	76
reduced sums disagree							
20 = 15+5	107	8, 17	26, 62	8	2	8, 17	20
23 = 8+8+7	163	10, 37	55, 91	1	5	10, 19	23
11+1 = 10+2	97	16, 25	43, 25	7	3	16, 7	12
16+3 = 12+7	152	26	71, 53	8	1	17	19
12+10 = 11+11	126	27, 18	36, 54	9	4	9, 18	22
17+7 = 12+12	174	21, 30	75, 48	3	6	21, 12	24
19+6 = 18+7	138	21	66, 75	3	7	12, 21	25
33+3 = 25+11	212	23	95, 77	5	9	23, 14	36
34+3 = 20+17	216	18	90, 72	9	1	18	37
45+4 = 30+19	186	15	78	6	4	15	49
47+2 = 31+18	207	27, 18	72, 99	9	4	18	49
47+3 = 39+11	205	25, 16	88	7	5	25	50
16+4 = 14+6	156	21, 12	66	3	2	12	20
21 = 20+1	141	6, 15	42	6	3	6, 15	21
22+11 = 21+12	228	21	66	3	6	21, 12	33

No coincidences of any sort were found in the non-agreeing group, not even in the tautologies! But two of the agreeing cases besides the anagram did show one other coincidence (second 2+2, first 4+4) and so were quadruply truthful, and seven more showed asymmetric or 'half' coincidences. The only other agreeing 2+2, EIGHTEEN + THREE = TWELVE + NINE, showed a coincidence that the anagram lacks (in lines 8 v 22). And it had as many full coincidences as the anagram in this table (three) *if* two 'halves' together be counted as one. Otherwise only the anagram was quintruply truthful, showing more than one other full coincidence. These results rule out uniqueness for some of the anagram's individual coincidences but their rarity and plurality do confirm the overall uniqueness of the anagram results.

In addition, not shown, I checked the reduced sums (#15 v. 19) of every tautological or redundant anagram equation of seven to eight digits whose reduced numeric sum (#19) was 4 as with 11+2= 12+1. There were seven such three-term tautologies (31=30+1, 49=40+9, 58=50+8, etc) and only one such redundancy (92+11=91+12). None had a reduced alpha of 4, ie none was triply truthful. Furthermore none of the other five 6-digit tautological anagrams (14+7=17+4, 14+9=19+4, etc) had reduced sums of 4 nor agreement between their own reduced sums.

This study showed in six ways that the anagram's coincidences are real and uncommon: (1) the small number of four term doubly truthfuls (n=n *and* a=a) out of several hundred equations examined; (2) of the doubles, the small (random?) number that were triply truthful (agreeing in reduced sums); (3) of the latter, the near absence of quadruply truthfuls and (4) the complete absence of other quintruply truthfuls (having more than one additional full coincidence); (5) the failure of all relevant redundant or tautological anagram equations to violate argument 2; and (6, previous section) the failure of other alphabet registers to duplicate the A=1 result.

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In addition to their value as "controls" for the anagram coincidences, all doubly truthful equations are logologically interesting in themselves, especially those that are triply or quadruply truthful. Now who's going to find a triply *equal* equation (n=n, a=a, n=a)? Or is there one?