PALINDROMIC WORD SQUARES

REX GOOCH
Letchworth Garden City, Herts, England
rexgooch@ntlworld.com

Introduction

A word square (n-square) consists of n rows, each containing a different n-letter word, so that the first column has the same word as the first row, and so on. A palindromic square is a word square in which the words are palindromes, with as many as possible being different. The investigation reported below was inspired by Zoran Radisavljevic's article (W05-178).

Let us construct a 7-letter square using as many different letters as possible. We can write ABCD for the first word before the rest of the word (CBA) is forced by the palindromic property. This word becomes the first column (word square property). The second row must begin with B, and we can continue EFG, with the rest of the word (FEB) being forced:

A B C D C B A
B E F G F E B
C F H I H F C
D G I J I G D
C F H I H F C
B E F G F E B
A B C D C B A

Generalising, we find these properties of palindromic word squares:

1. For *n*-letter words, the number of words in the square is ½ *n*, rounded up, at most. Thus there are just four different words in the above square.

2. The number of different letters is limited, to 4 + 3 + 2 + 1 = 10 in the case of the 7- and 8-squares, 15 in the case of 9- and 10-squares, and so on, the formula being ½ $h^*(h + 1)$, where *h* is the half length rounded up to the next integer: eg half of 7 rounded up is 4, and ½ $4^*(4 + 1)$ = 10. The number of letters in a square is n^2, so the ratio of different letters to all letters for even-length squares is ½ $h^*(h + 1)$ divided by n^2. As in this case *h* is exactly half of *n*, the ratio is 1/8 + 1/(4*n*). This is at a minimum of 12.5% when *n* is huge (realistically say 15% when *n* = 10) , and at its largest (sensibly speaking) at 18.75% when *n* = 4. This is not a great difference: the great majority of letters in palindromic squares are repeats. As the following table shows, when we include the values for odd-length squares, the percentage of letters that can be different is still around 15-20:

	no. of letters	maximum no. of different letters	%
6-square	36	6	17
7-square	49	10	20
8-square	64	10	16
9-square	81	15	19
10-square	100	15	15
11-square	121	21	17

3. A palindromic word square consists of a square element, repeated with its inversion, reflection, and inversion/reflection. In the square above, the square element defined by its side ABCD is necessarily repeated four times. Because the above example has an odd size, the four elements share a row and a column. These facts appear to lead to an easy way of making large squares. To make a palindromic 8-square, for example, one merely has to make a 4-square of 4-letter sequences, each being the start of a 8-letter palindrome. This is reminiscent of the efforts to

make 10-squares in the 1920s by using tautonyms. It is, indeed, very quick — but quick merely to show that there is an insufficient number of such specialised words.

4. Further, the basic square element itself consists of a triangle (ABCD/EFG/HI/J), so the whole square consists of a triangle repeated eight times, though squares with an odd number of rows will have triangles which share some letters. Thus, a given letter may be forced to appear eight times. In fact, in the above triangle, B, C and F each appear 8 times; A, D, E, G, H, and I appear four times each; and only J escapes replication. The case of the 8-square is simpler, as the basic square is repeated four times with no sharing, so all letters appear either four or eight times: A, E, H, and J appear four times, and the rest eight times.

5. The diagonals are identical, and palindromic

Summary of Results

Word length	Not place names words	Not place names squares	Place names words	Place names squares	Total words	Total squares	No. of squares with max # different letters
4	142	1143	65	289	207	2523	2058
5	1486	486054	690	119540	2176	1994231	878231
6	471	27964	174	4045	645	87877	7640
7	660	26504	218	3700	878	186843	314
8	162	41	8	0	170	49	0
9	308	782	27	0	335	1057	0
10	67	0	2	0	69	0	0
11	50	0	2	0	52	0	0

The table above divides palindromic words of a given length into those which certainly can be used as place names (even if they are also plain words), and those which are not likely to be place names; and shows the number of squares that can be made from each set, and the number that can be made from a combined set. It also shows the number of squares having the maximum number of different letters, as given in the first table in this article.

Among other things, the table shows that there are consistently far more words than place names; that squares are more difficult to find at length 4 (too few words) and at lengths 8 and above (too many words to combine); that I found no squares above length 9; and I found no "heterogrammatic palindromic squares" above length 7. The 7-square numbers are much larger than Radisavljevic reported. His article has no squares with the maximum number of letters.

Squares where the diagonals form a word are plentiful in the case of smaller squares, but do not exist above length 7. The total number of diagonal squares is broken down below into three components: those where the diagonal consists solely of the repeated letter A, those where the square has the maximum number of different letters, and others. There are no diagonal palindromic 8- or 9-squares.

Word length	Number of squares	No. of diagonal squares	Diagonal squares with max letters	Other diagonal squares	Diagonal squares with AAA… diags
4	2523	843	406	415	22
5	1994231	178305	44150	132882	1333
6	87877	4162	1	4029	132
7	186843	979	0	532	447

In all the following sections, the squares are made from a mixture of place names and ordinary words, though selection will favour non-place names. In selecting the best squares, words which occur as headwords in major dictionaries count as superior, but in any case expressions are

valued if they appear in everyday communications. Diagonal words are inset. To save space, only the top half of each square is shown, and sourcing has been mainly constrained to the OED and Palindromicon. Place names are from the NIMA database. W2 = Webster's Second. O = OED. Pa = Palindromicon.

Palindromic 4-squares

Best 4-squares with maximum number of different letters (3)

All words are OED headwords except SCCS (= Source Code Control System, 1972, Marc Rochkind at Bell Labs; *or* Skunk Creek Computing Services, Boulder, Colorado. In common use.), ISSI (Pa), ALLA (W2). All are in the most-used 80,000 expressions.

```
ABBA                ALLA    ANNA    ATTA    BEEB
BEEB or BOOB        LEEL    NOON    TOOT    ELLE or EMME or ESSE

BOOB                DEED                    ESSE    ISSI
OPPO or OTTO        ELLE or EMME or ESSE    SCCS    SCCS or SEES

LEEL                NOON            OPPO    PEEP
EMME or ESSE        OPPO or OTTO    PEEP    ELLE or EMME or ESSE

POOP    SEES            TOOT
OTTO    ELLE or EMME    OTTO
```

Best 4-squares regardless of number of letters

All words are OED headwords except SCCS (see above), MMMM (Pa), and DDDD (4D is a Canberra based company). All are in the most-used 84000 expressions.

```
AMMA    DDDD    ELLE    EMME    ESSE          LEEL    OPPO
MMMM    DEED    LEEL    MMMM    SCCS or SEES  ELLE    POOP

OTTO    POOP    SEES    TOOT
TOOT    OPPO    ESSE    OTTO
```

In diagonal 4-squares, the diagonal may be the same as one of the rows, thus:

starting with	ABBA	if A=B	AAAA	if B=C	ABBA	if A=C	ABBA
	BCCB		ABBA		BBBB		BAAB
	BCCB		ABBA		BBBB		BAAB
	ABBA		AAAA		ABBA		ABBA
diagonal same as:	neither		row2		row1		neither

Best diagonal 4-squares with maximum number of different letters (3)

```
ABBA O          AEEA Pa         ANNA O          ATTA O          ESSE O
BEEB O          EMME O          NEEN O          TEET O          SCCS (see above)
  AEEA Pa         AMMA O          AEEA Pa         AEEA Pa         ECCE O

ESSE O          SEES O          SEES O          TEET O          TSST Pa
SGGS Pa         ECCE O          EGGE O          ESSE O          SEES O
  EGGE O          SCCS (see above)  SGGS Pa         TSST Pa         TEET O
```

Best diagonal 4-squares regardless of number of letters

```
AAAA Pa         AAAA Pa         ABBA O          ATTA O          BBBB Pa
ABBA O          ANNA O          BBBB Pa         TTTT*           BEEB O
  ABBA O          ANNA O          ABBA O          ATTA† O         BEEB O
```

*Teaching Technology Through Tradition - see Internet †also Mohammed Atta

```
BOOB O        DEED O        EEEE Pa       ELLE O        ESSE O
OBBO O        EDDE Pa       ELLE O        LEEL O        SEES O
  BBBB Pa       DDDD*         ELLE O        EEEE Pa       EEEE Pa
```

*4D is a Canberra based company specialising in military spatial information systems.

```
KOOK O        MAAM O        NAAN O        NOON O        OBBO O
OOOO Pa       AMMA O        ANNA O        OOOO Pa       BOOB O
  KOOK O        MMMM Pa       NNNN Pa       NOON O        OOOO Pa

OPPO O        OTTO O        PEEP O        SEES O
POOP O        TOOT O        EEEE Pa       ESSE O
  OOOO Pa       OOOO Pa       PEEP O        SSSS*
```

*quadruplex SSSS = type of zygote in the tetraploid (OED quadruplex)

Palindromic 5-squares

Best 5-squares with maximum number of different letters (6) (all are OED headwords)

```
LEMEL         LEMEL         MAPAM         NAMAN         NEMEN
EBIBE         ERORE         ALULA         ALULA         ERORE
MINIM         MODOM         PUTUP(put-up) MUTUM         MODOM

NETEN                       RENER           SENES       TENET
EBIBE                       EBIBE           EBIBE       EBIBE
TIMIT or TIPIT   NIKIN*       NIKIN       NIKIN*
```

```
VENEV         WEREW or ZEREZ
EBIBE         EISIE
NIKIN*        RSFSR
```

* *or* NISIN. RSFSR = Rossíiskaya Sovétskaya Federatívnaya Sotsialistícheskaya Respúblika

Best 5-squares regardless of number of letters

The following words are all in the most used 90,000 English words.

```
KARAK Pa                    MALAM Pa      SALAS O
ABABA Pa or ARARA W2        as left       as left
RADAR O                     LAVAL O       LAVAL O
```

Best diagonal 5-squares with maximum number of different letters (6):

```
MARAM O       MUTUM O       SEDES O       SELES O
AIEIA‡        UAIAU*        EBIBE O       EBIBE O
RENER O       TIPIT O       DIOID†        LIOIL†
  MINIM O       MAPAM O       SBOBS O       SBOBS O
```

*Corrego Uaiau, Brazil, -16°45, -55°02 †Dioid algebra, Lioil Merrett - see WW 2003-114
‡Association internationale des Ecoles et Instituts d'Administration (see Internet)

```
SERES O       STATS O       STATS O
EBIBE O       TENET O       TOROT O
RIOIR Pa      ANDNA Pa      ARKRA†
  SBOBS O       SEDES O       SOKOS O
```

†Arkra Inc. - see WW 2003-114

Best diagonal 5-squares regardless of number of letters

```
MARAM O       MARAM O       NETEN O       NEUEN O
AAAAA Pa      AOAOA Pa      EIEIE Pa      EELEE Pa
RADAR O       RADAR O       TEKET O       ULULU O
  MADAM O       MODOM O       NIKIN O       NEUEN O
```

```
RENER O          REUER O          SERES O          TENET O
ESESE Pa         EELEE Pa         EISIE O          EIEIE Pa
NEFEN O          ULULU O          RSFSR O          NEMEN O
  RSFSR O          REUER O          SIFIS Pa         TIMIT O

TIMIT O
IEIEI Pa
MINIM O
    TENET O
```

Palindromic 6-squares

Best 6-squares with maximum number of different letters (6):

```
MARRAM O
ALEELA Pa     or AREERA Pa
REDDER O      or RENNER O

MODDOM O                  PULLUP O     RATTAR O
OLEELO Pa                 UREERU Pa    ABIIBA Pa or ADIIDA Pa
DENNED O or DESSED O      LEFFEL O     TIPPIT O

RATTAR O     RETTER O     SIRRIS O or IMAAMI Pa or INAANI Pa
ALEELA Pa    ESAASE Pa    ILAALI Pa
TENNET O     TALLAT O     RATTAR O

TALLAT O    WORROW O    WORROW O
AREERA Pa   ODAADO Pa   OLEELO Pa
LEFFEL O    RATTAR O    REDDER O or RENNERO or REPPER O or RETTER
```

Best 6-squares regardless of number of letters

The best squares are the MARRAM squares above.

Best (only!) diagonal 6-square with maximum number of different letters(6)

```
SIDDIS Pa
IRAARI Pa
DAOOAD Darya-i-Daøoad, Afghanistan, 37°17, 69°27
  SROORS Pa
```

Best diagonal 6-squares regardless of number of letters

```
MAMMAM O     MARRAM O                    REDDER O     RETTER O
AAAAAA Pa    AAAAAA Pa                   EEEEEE Pa    EEEEEE Pa
MARRAM O     RARRAR Pa or RATTAR O       DEDDED Pa    TENNET O
  MARRAM O     MARRAM O or MATTAM Pa       REDDER O     RENNER O

REDDER O                  RENNER O                    TERRET O
EEEEEE Pa                 EEEEEE Pa                   EEEEEE Pa
DENNED O or DEPPED Pa     NEDDEN Pa or NENNEN Pa      RENNER O
  RENNERO or REPPER O       REDDER O or RENNER O        TENNET O


RERRER Pa
EEEEEE Pa
REDDER O or RENNER O
  REDDER O or RENNER O
```

Palindromic 7-squares

Best 7-squares with maximum number of different letters (10):

DEVOVED O
ELIKILE Pa
VITATIV Pa
OKANAKO *or* OKARAKO *or* OKASAKO (all Pa)

DRAKARD Pa *or* PRAKARP Pa
REVIVER O
AVONOVA Pa
KINGNIK Pa

MARGRAM Pa
ATENETA Pa
REVIVER O
GNIDING Pa *or* GNIPING Pa

NORGRON Pa
OTELETO Pa
REPAPER O
GLADALG*

ROTATOR O
OSENESO Pa *or* OSENESO Pa
TEJIJET Pa
ANILINA Pa *or* ANIMINA Pa

*Gladalg, Australia, -27°28, 148°39

2 squares (not 4) from:

ROTATOR O
OSENESO Pa *or* OSENESO Pa
TEJIJET Pa
ANIHINA Pa *or* ANIKINA Pa

ROTATOR O
OSENESO Pa
TEJIJET Pa
ANILINA Pa *or* ANIMINA Pa

SARORAS Pa
AMEDEMA Pa *or* ATENETA Pa
REVIVER O
ODINIDO Pa *or* ONILINO Pa

SORBROS Pa
OTELETO Pa
REPAPER O
BLAQALB Pa

ADINIDA W2 *or* ODINIDO Pa
DYSESYD Pa
ISKUKSI Pa
NEUQUEN Pa

Best 7-squares regardless of number of letters

CINENIC O
IBAAABI Pa
NAURUAN O
EARLRAE Pa

OGOPOGO O
GATETAG Pa
OTTETTO*
PEEWEEP O

*Grove: Dictionary of Music: *Octet*, or *Ottett* (*Ottetto*)

and 5 squares from:

HADADAH O
ACECECA *or* AGELEGA *or* AMEDEMA *or* AMELEMA *or* ATELETA, all Pa
DEVOVED O *or* DEEDEED O
ACOSOCA Pa *or* ALOKOLA Pa *or* ADDADDA Pa

Best diagonal 7-squares

HADADAH O
ADIAIDA Pa
HAALAAH Pa
SAILIAS Pa

HADADAH O
ADIAIDA Pa
NAADAAN Pa, *or* NAASAAN Pa
SAIDIAS Pa, *or* SAISIAS*

SHEBEHS Pa
HADADAH O
EDICIDE Pa
BACKCAB Pa
SAIKIAS Pa

*1878 Browning, *La Saisias*

SRESERS Pa
REPAPER O
EPEREPE*
SARDRAS Pa
SEEDEES Pa

SRETERS Pa
REPAPER O
EPEREPE*
TARHRAT Pa, *or* TARTRAT Pa
SEEHEES Pa, *or* SEETEES Pa

*Rio Eperepe, Mozambique, -14°43, 38°46

Palindromic 8-squares

Best 8-squares

DAQIIQAD Pa
AMASSAMA Pa *or* ANASSANA Pa *or* AVASSAVA Pa
QAMAAMAQ Pa
ISATTASI Pa

DOORROOD Pa
OOOOOOOO Pa
OOCOOCOO Pa *or* OOROOROO Pa
ROODDOOR Pa

ELARRALE*
LIVEEVIL Pa
AVATTAVA Pa
RETEETER Pa *or* RETOOTER Pa

*El Arrale, Somalia, 2°58, 46°33

NORAARON Pa
OKERREKO Pa
REPOOPER Pa *or* RETOOTER Pa
AROLLORA Pa

OOROOROO Pa
OKERREKO Pa
REDEEDER *or* REPEEPER *or* RETEETER (all Pa)

OREMMERO Pa *or* ORENNERO Pa

SLEFFELS Pa
LEERREEL Pa
EEEEEEEE Pa *or* EELEELEE Pa
FREDDERF Pa

DEDEEDED Pa
ELARRALE G4p
DAQIIQAD Pa
ERICCIRE Pa

DREPPERD Pa
REDEEDER Pa
EDIEEIDE Pa
PEEPPEEP Pa
LOOPPOOL Pa
OOOOOOOO Pa
OOCOOCOO Pa *or* OOROOROO Pa
POOPPOOP Pa

ROODDOOR Pa
OOOOOOOO Pa
OOCOOCOO Pa *or* OOROOROO Pa
DOORROOD Pa

SAASSAAS Pa
AJATTAJA Pa
AAAAAAAA Pa
STANNATS Pa *or* STARRATS Pa

SNIFFINS Pa
NEILLIEN Pa *or* NOILLION Pa
IITOOTII Pa
FLOWWOLF Pa

Palindromic 9-squares

Best 9-squares

5 squares from:
ASSAMASSA Pa
SHALALAHS Pa *or* SHANANAHS Pa *or* SHARARAHS Pa
SARALARAS Pa
ALANANALA *or* ANANANANA *or* ANASASANA *or* ARAKAKARA *or* ARARARARA all Pa
MALAYALAM O

SAMASAMAS Pa
AKAKAKAKA Pa
MALAYALAM O
AKAHAHAKA Pa
SAYAPAYAS Pa *or* SAYASAYAS Pa

SAMASAMAS Pa
ALANANALA Pa
MALAYALAM O
ANANANANA Pa *or* ANASASANA Pa
SAYAPAYAS Pa *or* SAYASAYAS Pa

SAMASAMAS Pa
AMAMAMAMA Pa
MALAYALAM O
AMARARAMA Pa
SAYAPAYAS Pa *or* SAYASAYAS Pa

SAMASAMAS Pa
ANANANANA Pa
MALAYALAM O
ANASASANA Pa
SAYAPAYAS Pa *or* SAYASAYAS Pa

SARALARAS Pa
ANONANONA Pa
ROTAVATOR O
ANANANANA Pa *or* ANASASANA Pa
LAVALAVAL Pa *or* SAVASAVAS Pa

4 squares from:

SAMASAMAS Pa
AMARARAMA Pa
MALAYALAM Pa
ARAKAKARA Pa *or* ARARARARA Pa
SAYAPAYAS Pa *or* SAYASAYAS Pa

SAMASAMAS Pa
ARARARARA Pa
MALAYALAM O
ARAKAKARA Pa
SAYAPAYAS Pa *or* SAYASAYAS Pa

SARALARAS Pa
ARORARORA Pa
ROTAVATOR O
ARAKAKARA Pa *or* ARARARARA Pa
LAVALAVAL Pa *or* SAVASAVAS Pa

SAMASAMAS Pa
ARAKAKARA Pa
MALAYALAM O
AKAHAHAKA Pa *or* AKAKAKAKA Pa
SAYAPAYAS Pa *or* SAYASAYAS Pa