

MAGIC SQUARE LOGOLOGY

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A magic square, well-known to recreational mathematicians, consists of a set of numbers (integers) arranged in a square, such that the sum of each row and column and both of the principal diagonals is the same. Although magic squares date back thousands of years and have an extensive literature (see for example *Magic Squares and Cubes* by W.S. Andrews, published by Dover in 1960), the following logological problem involving magic squares appears to be new: construct a magic square in which the number-names in each row, column and diagonal share a letter (a different one in each case). Since there are an infinite number of solutions to this problem (just as there are an infinite number of magic squares), it is necessary to impose the further condition that the sum of the numbers in the square be as small as possible.

We restrict our study to 3-by-3 magic squares, leaving 4-by-4 or larger ones as an exercise for the computer-literate reader. The 3-by-3 square has the following form, where x , y and z are integers greater than or equal to zero (Andrews, p 129):

$$\begin{array}{ccc} x+2y+z & x & x+y+2z \\ x+2z & x+y+z & x+2y \\ x+y & x+2y+2z & x+z \end{array}$$

Note that the sum of all the numbers is nine times the central number.

The 3-by-3 magic square with smallest sum (45) is given at the left below. Clearly, this does not satisfy the logological constraints, but the square with a sum of 315 shown at the center does (can a magic square with a smaller sum be found?). The three number-names in the rows contain O, V (or F) and E, respectively; the three number-names in the columns contain I, H and Y; the three number-names in the two diagonals contain R and T.

As noted in the August Kickshaws, Eric Angelini proposed a less-restrictive problem: for *any* 3-by-3 square of numbers, arrange to have a common letter in each row, column and diagonal (different in each case). William Rex Marshall's solution totaled 215; can the one at the right below, with a sum of 139, be bettered? (If the diagonal requirement is waived, Dave Morice found a square with a sum of only 57.)

$$\begin{array}{ccc} 8 & 1 & 6 \\ 3 & 5 & 7 \\ 4 & 9 & 2 \end{array} \qquad \begin{array}{ccc} 32 & 31 & 42 \\ 45 & 35 & 25 \\ 28 & 39 & 38 \end{array} \qquad \begin{array}{ccc} 5 & 17 & 7 \\ 24 & 20 & 27 \\ 15 & 8 & 16 \end{array}$$

In "Alphamagic Squares" in the May 1991 Word Ways, Lee Sallows considered a closely-related problem: find a magic square in which the length of the number-names forms another magic square. The magic square at the left (sum 135), converts to the magic square at the right (sum 63).

$$\begin{array}{ccc} 5 & 22 & 18 \\ 28 & 15 & 2 \\ 12 & 8 & 25 \end{array} \qquad \begin{array}{ccc} 4 & 9 & 8 \\ 11 & 7 & 3 \\ 6 & 5 & 10 \end{array}$$