DOUBLE WORD SQUARES FROM SCRABBLE® TILES

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Here's a pretty problem, which I heard of recently from Scrabble player Eric Harshbarger (who heard it from another player, Jeff Myers, who says the problem has been kicking around for a few years): arrange the set of 100 Scrabble® tiles to make four 5x5 double word squares with all 40 words taken from one of the standard Scrabble word lists such as the *Official Scrabble Player's Dictionary*.

Not knowing whether a solution existed or not, I did some preliminary calculations. From Eric Harshbarger I obtained a list of the 8938 five-letter words in the new (2006) edition of the *Official Tournament and Club Word List* (sometimes called TWL06) and wrote a computer program to find all 5x5 double word squares constructible from these words – not paying attention to letter distribution at this point. Now, one way to proceed from there would be to try all combinations of four of these squares and see which ones match the letter distribution of the Scrabble tiles, allowing the two blanks to be assigned in any way. Unfortunately, there are about 10,000,000 double word squares, so this involves checking 10,000,000-choose-4 (on the order of 10^{27}) combinations, which is not feasible on a home computer.

So I decided to try an approach which sometimes works on problems like this: to make the problem easier, make it harder. That is, impose some additional restriction on the solution - one which hopefully does not drastically reduce the possibility of a solution, but which *does* drastically decrease the difficulty of searching for one. The arbitrary condition I imposed was to allocate the four difficult letters (J, Q, X and Z), which have to appear somewhere, one to each 5x5 square.

This reduces the combinations significantly, as there are "only" about 15,000 Q squares, 135,000 J squares, 425,000 Z squares, and 602,000 X squares. The product of these numbers is still too large for an exhaustive search, but I hoped that by some sort of successive-refinement heuristic search I might be able to find at least one solution. My first attempt failed miserably and had to be scrapped; my second approach was better, and managed to find a near-miss solution that would work if I could change two of letter tiles into blanks (for 4 blanks total). Encouraged by this I made a few more tweaks and set it running again. Only ten minutes later the first solution came up on the screen:

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QUILT   FJORD   BATCH   AGATE
UNMEW   LAVER   IVORY   WAFER
ORIBI   ELITE   MIXUP   AZONS
TINES   YONIC   ASIDE   ROUGE
AGENT   SPEAK   HONED   DOLES
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Just by luck this solution happens to have the two blank tiles (shown as underlined letters) in the corner of one of the squares, which seems aesthetically nice.

Was this first find just exceptionally lucky? Not really. I've let the program run for a total of 20 hours, and so far it has found one additional solution about every ten minutes (for a total of 121 so far). Here are two more solutions having a fair number of common words:

CHEVY	FOIST	GAULT	BRAWL
ROQUE	JUNTA	AMNIA	RADII
ALULA	OPTIC	TIMER	OZONE
FEIGN	RHINO	OGIVE	WEBER
TYPOS	DESKS	RAXED	NEEDS

QUAFF	HOTCH	SWEPT	GOLEM
UNBAR	IVORY	PIXIE	AZURE
AWAKE	JORUM	OVINE	DONGA
DICER	ALIEN	RENAL	INGOT
STADE	BOILS	TRESS	DEITY

All the solutions shown here also have the property that all 40 words are different. Most solutions have this property anyway, so I decided to make it a requirement.

Do the new words in TWL06 (as compared the previous edition, TWL98, from 1998) make all the difference? No, as Jeff Myers checked my 121 solutions and found that in 29 of them all forty words can also be found TWL98.

We can only guess how many solutions there are in all. I suspect that my current search program would, if left running, find hundreds more, and those wouldn't include any of the solutions in which J, Q, X, and Z are allocated differently among the four squares. It *is* possible to allocate them in other ways – there are lots of 5x5's containing two of {J,Q,X,Z}, and even a hundred or so squares which contain three of them (but none containing all four). These possibilities would surely lead to even more solutions.

Although four 5x5's is a very elegant configuration, the problem could be posed with other combinations of double word squares and/or rectangles. Obvious choices include (1) one 6x6 and four 4x4's, (2) two 3x3's, two 4x4's and two 5x5's, or (3) a 4x4, a 6x6, and two 4x6 rectangles. Readers are encouraged to tackle these variants.