# EMBEDDING MANY WORDS IN PI

A. ROSS ECKLER
Morristown, New Jersey

The digits of the irrational number pi (3.14159265358979323846 26...) not only go on forever but behave statistically like a sequence of random numbers uniformly distributed on 0,1,...,9; no matter how long it may be, any finite subsequence of digits will appear an infinite number of times. Logologists seek to transform pi into a similarly-random sequence of words. The most obvious way of doing this is to write pi in base 26 using the letters of the alphabet in place of the digits 0 through 9, or in base 27, adding a space so that the infinite letter sequence is broken into "words". Details can be found in Lee Sallows, "Base 27: The Key to a New Gematria" WW May 1993, and Mike Keith, "The Pi Code", WW Nov 1999. Although all the words in the dictionary (in fact, all the plays of Shakespeare) are repeated an infinite number of times, they are rather sparsely scattered. Base 26 pi begins D.DRSQLOLYTROD...which reveals LO, ROD and TROD; in fact, in the first 100 letters one also finds ME, MU, EX, BY, GO, IS, ISM, EL and QI. However, the first five-letter word, STEEL, does not appear until position 6570, OXYGEN appears at position 11582, and SUBPLOT at position 115042.

Can one convert pi to a denser stream of words? Consider the following transformation of digits to letters: for any run of consecutive pi-digits, find their sum S and convert this to a letter of the alphabet using Z=0, A=1,...Y=25 (except for Z, this preserves the standard letter count).If the consecutive pi-digits sum to more than 25, reduce it to the range 0 to 25 by successive subtractions of 26 (S mod 26). For example, 3.1=4 becomes D, 415926=27 becomes A, and 535=13 becomes M. Words can then be formed by combining adjacent runs, so that 3.1415926535 becomes DAM.

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | C | A | D | A | E | I | B | F | E | C | E | H | I | G | I | C | B | C | H | D | F | B | F | D | C | C | H |
| 2 | D | E | E | F | N | K | H | K | H | H | M | Q | P | P | L | E | E | K | L | J | H | H | J | G | F | K | K |
| 3 | H | F | J | O | P | Q | M | N | M | P | V | X | Y | S | N | H | M | O | R | L | N | L | M | J | N | N | M |
| 4 | I | K | S | Q | V | V | P | S | U | Y | C | G | B | U | Q | P | Q | U | T | R | R | O | P | R | Q | P | T |
| 5 | N | T | U | W | A | Y | U | A | D | F | L | J | D | X | Y | T | W | W | Z | V | U | R | X | U | S | W | C |
| 6 | W | V | A | B | D | D | C | J | K | O | O | L | G | F | C | Z | Y | C | D | Y | X | Z | A | W | Z | F | H |
| 7 | Y | B | F | E | I | L | L | Q | T | R | Q | O | O | J | I | B | E | G | G | B | F | C | C | D | I | K | H |
| 8 | E | G | I | J | Q | U | S | Z | W | T | T | W | S | P | K | H | I | J | J | J | I | E | J | M | M | K | J |
| 9 | J | J | N | R | Z | B | B | C | Y | W | B | A | Y | R | Q | L | L | M | R | M | K | L | S | R | N | M | R |
| 10 | M | O | V | A | G | K | E | E | B | E | F | G | A | X | U | O | O | U | U | O | R | U | R | X | P | U | Z |
| 11 | R | W | E | H | P | N | G | H | J | I | L | I | G | B | X | R | W | X | W | V | A | Z | X | T | X | C | D |
| 12 | Z | F | L | Q | S | P | J | P | N | O | N | O | K | E | A | Z | Z | Z | D | E | F | Z | Z | B | F | G | E |
| 13 | I | M | U | T | U | S | R | T | T | Q | T | S | N | H | I | C | B | G | M | J | F | B | H | J | J | H | N |
| 14 | P | V | X | V | X | A | V | Z | V | W | X | V | Q | P | L | E | I | P | R | J | H | J | P | N | K | Q | U |
| 15 | Y | Y | Z | Y | F | E | B | B | B | A | A | Y | Y | S | N | L | R | U | R | L | P | R | T | O | T | X | V |
| 16 | B | A | C | G | J | K | D | H | F | D | D | G | B | U | U | U | W | U | T | T | X | V | U | X | A | Y | B |
| 17 | D | D | K | K | P | M | J | L | I | G | L | J | D | B | D | Z | W | W | B | B | B | W | D | E | B | E | K |
| 18 | G | L | O | Q | R | S | N | O | L | O | O | L | K | K | I | Z | Y | E | J | F | C | F | K | F | H | N | N |
| 19 | O | P | U | S | X | W | Q | R | T | R | Q | S | T | P | I | B | G | M | N | G | L | M | L | L | Q | Q | W |
| 20 | S | V | W | Y | B | Z | T | Z | W | T | X | B | Y | P | K | J | O | Q | O | P | S | N | R | U | T | Z | F |
| 21 | T | X | C | C | E | C | B | C | Y | A | G | G | Y | R | S | R | S | R | X | W | T | T | A | X | C | I | I |

In the table above, each letter corresponds to a specific run of consecutive digits of pi; if it occurs in row i and column j, the run starts at the jth digit of pi and is i units long. For example, F in row 3 and column 2 corresponds to the run of length 3 starting at the second pi-digit: 141. In other words, the runs corresponding to the first row of letters are 3,1,4,1,5,9... the runs corresponding to the second row of letters are 31,14,41,15,59... and the runs corresponding to the third row are 314,141,415... Words can be read off from the table by selecting letters corresponding to abutting runs: A has a run of length one in the second position, D a run of length one in the third position, and O a run of length three in the fourth, fifth and sixth positions.

The score of a word is equal to the position of the last digit of pi needed to complete it; for example, ADO, corresponding to (3) 1 4 159, has a score of 6. The list below gives all the words in Webster's Pocket Dictionary with a score of 12 or less. Note that HE appears three times.

| | |
|---|---|
| 2 | A 1 |
| 3 | CAD 3 1 4, AD 1 4 |
| 4 | A 1 |
| 5 | |
| 6 | ADO 1 4 159, DO 4 159, AN 1 59, DO 1 4 159, I 9, AS 1 4159, IN 3141 59, DEN 31 41 59 |
| 7 | FIB 15 9 2, JIB 415 9 2, HAP 314 1 592 |
| 8 | CASH 3 1 4159 26 |
| 9 | WE 15926 5, THE 14159 26 5, HE 26 5, SHE 4159 26 5, BE 1415926 5 |
| 10 | NIP 31415 9 2653, HE 314 1592653, DAD 4 1 592653, FEY 141 5 92653, HAD 314 1 592653, DUN 31 41592 653 |
| 11 | ACE 59625 3 5, HE 53 5, DAM 31 415926 535, AM 415926 535, DEED 31 41 5 926535 CAN 3 1 415926535, FED 141 5 926535, DOPE 4 159 2653 5 |
| 12 | PA 592 65358, HEP 26 5 358, DIM 31 41592653 58 |

It is gratifying to see how quickly long words appear embedded in pi: CAD 3, CASH 8, EQUIP 15, CASHER 16 (DEMISE 18), CADAVER 16 (DIMPLED 24), SICKLIER 36 (CADAVERS 37), CADAVERIC 25, CADAVEROUS 44 (INSCRIBING 47), COMPETITIVE 65, EPIGLOTTISES 59

There are other ways of converting pi-digits to letters; perhaps the most obvious alternative is to regard a run of consecutive digits as a number in its own right, and reduce it to the range of 0 to 25 by successive subtractions of 26. For example, the sequence 141, instead of summing to 6 and producing F, yields the remainder of 11 after five subtractions of 26, corresponding to the letter K. As before, the construction of words is facilitated by a table of letters corresponding to all runs of pi-digits (see next page).

The words with scores of nine or less are:

| | |
|---|---|
| 2 | A 1 |
| 3 | CAD 3 1 4, AD 14 |
| 4 | A 1 |
| 5 | DO 4 15, ADO 1 4 15, NO 14 15, CAY 3 1 415 |
| 6 | NAG 14 1 59, CAY 3 1 4159, BAG 314 1 59 |
| 7 | KEN 141 5 92, DON 4 15 92, AT 1 592, ON 15 92, CAR 3 1 41592, BAT 314 1 592 |
| 8 | OX 41 5926 |
| 9 | DOPE 4 15 926 5, ARM 1 41592 65, AS 1 4159265, EYE 31 4159 265. |

ADAGE 1 4 1 59 265

Again, long words embedded in pi appear quickly: CAD 3, DOPE 9, ADAGE 9, ENFOLD 24, CARRIES 31, VERTICES 31, ANCHYLOSE 47 (BRIGADIER 51), EXCLOSURES 56 (ADDICTIONS 57), COLLOQUIZED and KINDHEARTED 75, ANNIHILATION 90.

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | C | A | D | A | E | I | B | F | E | C | E | H | I | G | I | C | B | C | H | D | F | B | F | D | C | C | H |
| 2 | E | N | O | O | G | N | Z | M | A | I | F | K | S | A | O | F | W | L | F | T | J | Z | L | Q | G | L | E |
| 3 | B | K | Y | C | T | P | E | C | O | T | Q | M | Q | M | V | K | D | T | N | T | B | D | S | Q | Z | S | Z |
| 4 | U | K | Y | F | X | I | A | I | B | A | U | I | Q | B | O | N | R | X | L | X | X | Q | K | V | C | J | G |
| 5 | G | O | R | N | K | O | O | T | C | Q | K | O | P | W | B | N | D | H | V | J | I | Q | N | O | F | C | A |
| 6 | A | V | D | O | I | Y | B | A | K | W | I | V | G | D | X | P | P | H | P | Y | O | V | M | V | O | M | O |
| 7 | L | R | S | W | Q | X | C | Q | O | Y | N | O | Z | R | L | F | J | F | G | S | B | O | B | S | C | E | T |
| 8 | V | C | K | A | V | O | K | W | W | R | M | B | D | D | R | N | Z | K | U | P | W | V | A | Q | I | X | T |
| 9 | Q | G | K | R | U | A | O | Y | X | A | H | X | T | P | D | N | C | I | J | G | X | S | S | S | L | H | Z |
| 10 | Q | W | N | G | I | S | W | R | I | R | F | L | T | J | R | M | G | T | Y | T | M | Q | M | H | R | J | H |
| 11 | S | D | S | Y | U | K | X | A | T | B | N | R | X | Z | A | C | Z | U | R | Y | I | S | Z | D | F | D | F |
| 12 | P | W | O | Y | E | H | I | R | V | Z | L | D | J | C | M | L | C | D | E | Y | Q | H | V | B | P | R | I |
| 13 | M | C | C | S | Z | E | T | B | R | B | V | R | Y | G | H | S | F | U | G | U | N | D | B | T | H | Y | U |
| 14 | G | M | G | J | C | F | V | Z | Z | Z | P | A | S | Z | E | J | O | K | W | B | L | B | V | B | C | Y | I |
| 15 | A | C | T | Y | L | L | R | B | F | D | G | M | P | C | Z | C | C | K | V | V | X | T | X | M | M | W | M |
| 16 | M | F | U | X | T | V | Z | Z | L | Q | U | H | G | F | G | M | I | F | N | T | N | V | G | I | G | W | F |
| 17 | B | K | J | J | X | N | F | D | S | Q | J | E | T | O | A | E | L | J | R | Z | N | M | A | S | S | B | Q |
| 18 | W | N | Z | B | H | P | L | Q | K | V | Y | Z | Y | C | O | X | R | D | F | D | K | I | Q | I | N | C | Q |
| 19 | D | N | F | V | H | H | S | Q | N | O | R | G | Y | I | T | H | F | V | L | O | O | S | O | R | S | G | W |
| 20 | R | P | J | R | F | E | K | V | M | V | E | A | U | L | T | J | P | P | Q | C | A | I | Z | G | K | A | E |
| 21 | D | F | B | B | K | A | N | O | B | S | G | O | B | R | Z | D | H | E | W | K | K | R | I | U | O | S | A |

Mike Keith points out that more words can be found early in the pi-sequence if one eliminates the requirement that adjacent letters have abutting pi-sequences.

In his weblog Logolog, Eric Harshbarger (Jan 13-14) suggests a different way of forming words from pi digits. For a word of length L, partition the first SL digits of pi, starting after the decimal point, into L slices of length S and reduce each slice mod 26 to a number between 0 and 25. Assign letters to these according to the rule A=0, B=1, … Z=25; is a word formed? If not, try again with slices of length S+1 until a word appears. The first word of length two that is revealed by this method is surprisingly early; BE requires a slice of length one! The first three-letter word to appear is EYE with a slice of 11:

14159265358 mod 26 = 4
97932384626 mod 26 = 24
26433832795 mod 26 = 4

However, longer words are much slower to first appear: WEIR with a slice of 206, CANAL with 600, WHARVE with 46441, ABLEIST with 89842, and OCCUPIER with 4722439. In other words, the first eight-letter word requires more than 37 million digits of pi to show up!

I am indebted to Mike Keith for supplying the two tables as well as the longer examples of embedded words.