

NUMBER NAMES WITH A SNOWBALL LETTER DISTRIBUTION

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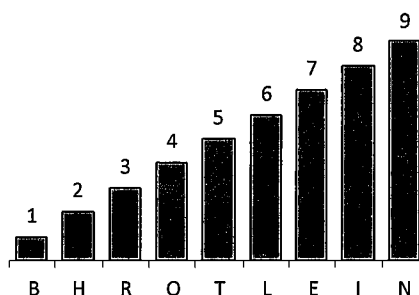
Introduction

The spelling out of numbers in English words is a frequently studied topic by logologists. For example, it has long been known that the numbers 0, 1, 2, 4, 5, 6, 8, 10, 40, 46, 60, 61, 64, 80, 84, and 5000 are the only ones whose written-out names are heterograms (no letter of the alphabet occurs more than once). Another interesting number is 1,001,000,000,001,000,001,001,002,568 which contains twenty-three different letters of the alphabet (the maximum, since J, K, and Z can never occur).

Pondering the general idea of letter distribution in a written-out number one evening, Harshbarger found, by hand, the following interesting number:

3,000,000,000,000,000,013,019,000,000,000

THREE NONILLION
THIRTEEN TRILLION
NINETEEN BILLION



As illustrated by the diagram on the right, the letter distribution in this number's name is a perfect snowball, consisting of 1 B, 2 H's, 3 R's, and so on. We call such a number a *snowball-histogram number*, or *SH number* for short. An SH number whose histogram values are 1...N is said to have *order* N.

Several questions naturally suggest themselves. For which orders do SH numbers exist? How many such numbers are there for each order? For a given order, what is the smallest and largest SH number? What is the largest and smallest SH number of *any* order?

Definitions and Conventions

We use the AND-less convention for writing number names, in which 573 is FIVE HUNDRED SEVENTY THREE, not FIVE HUNDRED AND SEVENTY THREE.

In a number name, the words which represent powers of 1000 (THOUSAND, MILLION, BILLION, etc.) will be called *power words*, while the phrases inserted between them (or following the last power word) that represent numbers 1 to 999 inclusive are the *modifiers*. Starting with a given SH number and permuting the modifiers in any fashion produces another SH number. We consider variations generated this way to be trivial solutions, and when presenting a solution we will typically sort the modifiers to produce the largest or smallest numerical value.

We decided to allow only the standard dictionary power words, which are those in the table under the entry for *number* in Webster’s 3rd Unabridged. These consists of powers of 1000 from 1 to 21 (THOUSAND, MILLION, BILLION,....., NOVENDECILLION, VIGINTILLION) plus CENTILLION, which is 1000 to the power 101. Note that we are using the “short scale” convention (sometimes called the “American system”) when we convert between numbers and number names. If one were to use the other, “long scale”, convention then our results expressed as number names would be the same but their representation as numbers would be different. What about the power words GOOGOL and GOOGOLPLEX? We decided not to include these for the bulk of this study, but later on we will give a few interesting results that arise from allowing them.

Results

We had no idea how many SH numbers there might be of the various orders, so Keith wrote a computer program to search for them. In a given run the program covers the search space for a specific order N, as opposed to searching for SH numbers of all orders simultaneously. This allows the search to be much faster because as it generates trial combinations of power words and modifiers it only needs to consider those that use N or fewer letters. We also know that an SH number name of order N will contain exactly $N(N-1)/2$ total letters, which leads to another significant optimization: as the program traverses the search tree, it can stop its forward progress immediately if it determines that concatenating even one more power word or modifier will cause it to exceed the $N(N-1)/2$ letter limit. Using this program we were able to exhaustively search up to order 12 and do a partial search up to order 16.

There are no SH numbers of order 7 or less. Below are all the SH numbers of order 8 and 9. Remember, as mentioned previously, that the modifier words in each of these can be permuted to yield additional numbers.

	1...8
BCTEOLIN	10 Cen + 9,000,000,000,000,001,000,000,000 TEN CENTILLION NINE OCTILLION ONE BILLION
MCTEOLIN	1 Cen + 9,000,000,000,000,000,010,000,000 ONE CENTILLION NINE OCTILLION TEN MILLION
	1...9
CHROTLIEN	19 Cen + 3,000,000,000,000,003,000,000,000,000 NINETEEN CENTILLION THREE NONILLION THREE TRILLION
DCTFOLEIN	15 Cen + 15,001,000,000,000,000,000,000,000,000 FIFTEEN CENTILLION FIFTEEN DECILLION ONE NONILLION
DFCOTLEIN	15 Cen + 10,000,010,000,000,000,000,000,000,009 FIFTEEN CENTILLION TEN DECILLION TEN OCTILLION NINE
DFCTOLEIN	15 Cen + 10,000,009,000,000,000,000,000,000,001 FIFTEEN CENTILLION TEN DECILLION NINE OCTILLION ONE
DWCETLION	10 Cen + 9,000,000,000,000,000,000,002,000,000,000,000,000,000,002 TEN CENTILLION NINE OCTODECILLION TWO NONILLION TWO
RDCTOLEIN	19,000,000,000,000,000,000,010,000,000,000,000,009,000,000,000,000 NINETEEN OCTODECILLION TEN DECILLION NINE TRILLION
DFCTELOIN	15,000,000,000,000,000,000,000,010,009,000,000,000,000,000,000,000,000 FIFTEEN OCTODECILLION TEN NONILLION NINE OCTILLION
DWCETLINO	19,000,000,000,000,000,000,002,002,000,000,000,000,000,000,000,000 NINETEEN OCTODECILLION TWO NONILLION TWO OCTILLION

BHROTLEIN	19,000,000,000,000,000,013,003,000,000,000 NINETEEN NONILLION THIRTEEN TRILLION THREE BILLION
MHROTLEIN	19,000,000,000,000,000,013,000,003,000,000 NINETEEN NONILLION THIRTEEN TRILLION THREE MILLION
BHROTLIEN	19,000,000,000,000,000,010,003,000,000,003 NINETEEN NONILLION TEN TRILLION THREE BILLION THREE
MHROTLIEN	19,000,000,000,000,000,010,000,003,000,003 NINETEEN NONILLION TEN TRILLION THREE MILLION THREE
BHRTOLIEN	19,000,000,000,000,000,003,003,000,000,001 NINETEEN NONILLION THREE TRILLION THREE BILLION ONE
MHRTOLIEN	1,000,000,000,000,000,003,000,003,000,019 ONE NONILLION THREE TRILLION THREE MILLION NINETEEN

Within each order the solutions are presented from largest to smallest and in each solution the modifiers are ordered to maximize the size of each number, except for the last entry under each order where the modifiers have been ordered to minimize the size of the number. This means that the first or last entry under each order is precisely the SH number having the largest or smallest numerical value.

In these lists each number is displayed above its number name, but since a centillion is so large we abbreviate the centillion component using an expression like “10 Cen +”. In the left column of each entry the letters used, in order of increasing frequency, are shown.

For orders 10 and above there are many hundreds or thousands of solutions, so we will just show the largest and smallest number for each order. An exhaustive search of orders 10-12 produces these results:

1...10	
Largest:	93 Cen + 30,000,000,000,000,000,013,000,000,000,001
CYHROLETIN	NINETY THREE CENTILLION THIRTY NONILLION THIRTEEN TRILLION ONE
Smallest:	3,010,000,000,000,000,033,000,000,000,099
CYHROLETIN	THREE NONILLION TEN OCTILLION THIRTY THREE TRILLION NINETY NINE
1...11	
Largest:	92 Cen + 51,000,000,000,000,000,000,000,050,000,001,000,000,000,000,000,000,000,000,000,000,000
WDYFCETLNOI	NINETY TWO CENTILLION FIFTY ONE OCTODECILLION FIFTY DECILLION ONE OCTILLION
Smallest:	3,010,030,000,000,000,000,000,030,000,000,000,093
DCYHROELNTI	THREE DECILLION TEN NONILLION THIRTY OCTILLION THIRTY TRILLION NINETY THREE
1...12	
Largest:	933 Cen + 10,000,000,000,000,010,010,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000
YUHRCDOLTINE	NINE HUNDRED THIRTY THREE CENTILLION TEN OCTODECILLION TEN TREDECILLION TEN DUODECILLION
Smallest:	1,000,000,000,000,000,030,000,333,000,399
MUYDOLHRTIEN	ONE NONILLION THIRTY TRILLION THREE HUNDRED THIRTY THREE MILLION THREE HUNDRED NINETY NINE

For orders 13 to 15 we were not able to do an exhaustive search in a reasonable amount of computer time but were still able to find the provably smallest solution for orders 13 and 14. In the other cases (those that say “smallest known” or “largest known”) the solution given is the best we found but it is not known to be maximal or minimal.

1...13

Largest known: 993 Cen + 340,000,000,000,000,000,000,000,030,003,000,000,000,000,000,000,000,000,000,000,000
FUYCDHRLTIEN NINE HUNDRED NINETY THREE CENTILLION THREE HUNDRED FORTY OCTODECILLION THIRTY NONILLION THREE OCTILLION

Smallest: 300,000,000,000,810,000,830,000,000,000,890
QYGOLURDHTEIN THREE HUNDRED NONILLION EIGHT HUNDRED TEN QUINTILLION EIGHT HUNDRED THIRTY TRILLION EIGHT HUNDRED NINETY

1...14

Largest known: 998 Cen + 333,000,000,000,000,000,000,230,000,000,000,000,000,000,000,000,000,000,000,000,000,200
GWYCULORHDITNE NINE HUNDRED NINETY EIGHT CENTILLION THREE HUNDRED THIRTY THREE OCTODECILLION TWO HUNDRED THIRTY UNDECILLION TWO HUNDRED

Smallest: 313,340,350,000,000,000,499
AQYFOLUHDITRNE THREE HUNDRED THIRTEEN QUINTILLION THREE HUNDRED FORTY QUADRILLION THREE HUNDRED FIFTY TRILLION FOUR HUNDRED NINETY NINE

1...15

Largest known: 999 Cen + 844,000,444,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,333
VGCYFLUHDOTRIEN NINE HUNDRED NINETY NINE CENTILLION EIGHT HUNDRED FORTY FOUR VIGINTILLION FOUR HUNDRED FORTY FOUR OCTODECILLION THREE HUNDRED THIRTY THREE

Smallest known: 300,000,310,330,000,000,446,000,000,000,669
PFYXSUOLHDTRINE THREE HUNDRED NONILLION THREE HUNDRED TEN SEPTILLION THREE HUNDRED THIRTY SEXTILLION FOUR HUNDRED FORTY SIX TRILLION SIX HUNDRED SIXTY NINE

The smallest solution for order 14, the number 313,340,350,000,000,000,499, is shockingly small compared to all other known SH numbers (of any order). It seems very likely that this is the smallest SH number of any order, but a proof of this fact, even with computer assistance, seems difficult. The largest known SH number of any order is the order-15 example shown above. Note that this number is 99.9844% of the largest number expressible using the standard dictionary names.

The search space for order 16 is so vast that we could only search a tiny fraction of it, and orders above that are even more enormous. By sheer luck we found one SH number of order 16:

988 Cen + 444,444,034,000,000,000,000,010,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000
(MVGYCHFUDLTRENOI)
NINE HUNDRED EIGHTY EIGHT CENTILLION FOUR HUNDRED FORTY FOUR VIGINTILLION FOUR HUNDRED FORTY FOUR NOVEMDECILLION THIRTY FOUR OCTODECILLION TEN TREDECILLION

The situation for SH numbers of orders 17 to 23 is unknown: we have not found any solutions, but neither is there a proof that such solutions do not exist.

If the power words GOOGOL and GOOGOLPLEX are allowed then it is possible to snag one additional order, with the following unique order-7 SH number:

VGILEON 11 googol + 9,000,000,000,000,000,000,000,000,001
ELEVEN GOOGOL NINE NONILLION ONE

And it is possible to obtain truly huge SH numbers, such as this order-13 example:

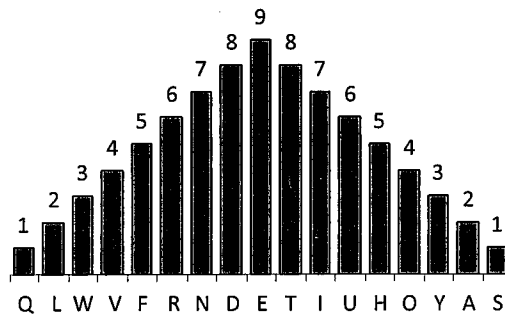
PVXSYHLNOGTIE 98 googolplex + 88 googol + 78,000,000,078,000,000,000,000,000,068
 NINETY EIGHT GOOGOLPLEX EIGHTY EIGHT GOOGOL SEVENTY EIGHT
 NONILLION SEVENTY EIGHT SEXTILLION SIXTY EIGHT

It is worth noting that GOOGOL by itself (with no modifier) is an order-3 SH word.

The idea we've explored in this article can be broadly generalized to any other type of interesting letter distribution. For example, here is a relatively small number of order 17 whose letter distribution is a growing/melting snowball:

224,000,000,000,525,535

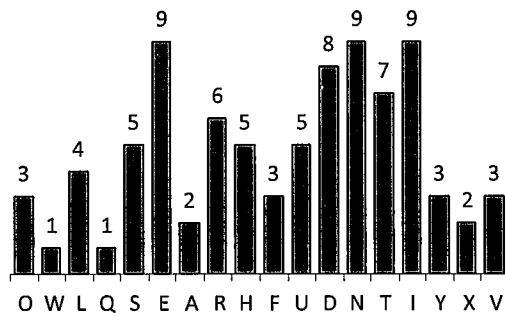
TWO HUNDRED TWENTY FOUR
 QUADRILLION
 FIVE HUNDRED TWENTY FIVE
 THOUSAND
 FIVE HUNDRED THIRTY FIVE



and here is one of order 18 whose histogram values are the first 18 digits of the number π :

520,636,000,000,757,000

FIVE HUNDRED TWENTY
 QUADRILLION
 SIX HUNDRED THIRTY SIX
 TRILLION
 SEVEN HUNDRED FIFTY SEVEN
 THOUSAND



This idea can also be applied to arbitrary text, not just number names. Can you find a sentence in *Moby Dick* or *Pride and Prejudice* whose letter distribution is a snowball or is interesting in some other way? Such possibilities are left for future consideration.