

TWO DIRECTED STATE NETWORKS

A. ROSS ECKLER

Morristown, New Jersey

In a directed network, one can move from node to node in one direction only. The set of 50 states form a directed network in either of two configurations.

In the first configuration, one moves from one state to the next if the last letter of the first state is the first letter of the next state, as wisconsiN-New york or rhode islanD-Delaware.

Ohio, New Mexico and Oregon form a small core, within which one can reach any state from any other. A second, slightly larger core consists of Alaska, Arizona, Arkansas, South Dakota and Carolina. Members of a core are called insiders. One can proceed from the first core to the second via Oklahoma, Nevada, North Carolina, North Dakota, Nebraska or the two-step New York and Kansas.

There are a number of states that feed into one or both cores, consisting of starters (states that cannot be reached from any other state) and precursors (states joining starters to a core). There are 21 of the former and 6 of the latter:

Utah-Hawaii-Illinois, Utah-Hawaii-Iowa, Utah-Hawaii-Indiana
Mississippi-Illinois, Mississippi-Iowa, Mississippi-Indiana
Missouri-Illinois, Missouri-Iowa, Missouri-Indiana
Vermont-Texas
Connecticut-Texas
Wyoming-Georgia
Massachusetts
Montana
Michigan, Michigan-North Carolina, Michigan-North Dakota
Minnesota
Wisconsin, Wisconsin-North Carolina, Wisconsin-North Dakota
Washington, Washington-North Carolina, Washington-North Dakota
West Virginia
Virginia
Florida
Pennsylvania
California
Colorado

A few starters cannot reach the core, but instead proceed to enders, states that cannot be followed by any other state. Enders include Delaware, New Jersey, Kentucky, New Hampshire, Tennessee and Maine. Starters going to enders only are Maryland to Delaware, Rhode Island to Delaware, and Maine. Starters going to enders as well as a core include Vermont to Tennessee and Connecticut to Tennessee. Since Maine is both a starter and an ender, it is in a network by itself.

Note that one can exit from the first core to enders New Hampshire, New Jersey and (via New York, a follower) Kentucky. However, once one is in the second core one cannot exit from it.

What state can access most other states? It appears to be Utah, which via Hawaii and Illinois-Iowa-Indiana reaches the 3 states in the first core, the 5 states in the second core, the 7 states connecting the first core to the second, and ends Kentucky and New Hampshire, a total of 21. What state can be accessed by the most states? Any of the 5 states in the second core can be reached by 45 others—the only ones that cannot reach them are Maryland, Delaware, Rhode Island and Maine.

Finally, what is the span of the network? If a beginner is linkable to an ender, one can ascertain the minimum number of links needed to do it; the span is the maximum of these numbers, taken over all linkable beginner-ender pairs. The span appears to be 5, achieved by Utah-Hawaii-Idaho-Oregon-New York-Kentucky.

In the second configuration, one moves from one state to the next if the last letter of the postal code for the first state matches the first letter of the postal code for the second state, as MD-DE.

The core of this directed network is much larger, consisting of 30 states: AL, AK, AR, CA, CO, CT, HI, IA, IL, IN, KS, NC, NH, NM, NV, LA, MI, MA, MO, MS, MT, MN, OH, OK, OR, RI, SC, TN, VA and VT. There are 7 starters that feed into the core: UT, FL, GA, PA, WA, WV and WI. In turn the core leads directly to the 8 enders NJ, NY, KY, WY, TX, AZ, NE ME and indirectly to the ender DE via followers ND, MD, ID or SD.

What state can access most other states? Any starter can reach 43 other states—all but the six other starters. Which state can be accessed by the most states? DE can be reached by 41—all but the eight other enders. The span of the network is almost certainly 7: FL-LA-AK-KS-SC-CO-OH-HI. One can also ascertain the round-trip span, the maximum number of steps necessary to go from one state in the core to a second state and then back to the first. I believe that the round-trip span is 10, achieved by going from NV to OK and return: NV-VT-TN-NC-CO-OK-KS-SC-CT-TN-NV. Are there any two states in the core which require at least 11 steps?

The Nov 1999 Word Ways contains my article “The Postal Union” which considered a hybrid version of the directed state network. In this version, one proceeded from one state to the next if the postal code of the second state appeared as a bigram in the name of the first state (as ND or IA in Indiana). In the main network, there were 20 starters and 8 preceders leading into a core of 8 states; in turn the core led to two of the 6 enders. Three states—Kansas, Texas, Kentucky—were simultaneously starters and enders, and thus independent of the main network. The span of this network was 9: verMOnt, MIssouri, micHIgan, haWAii, washINgton, iNDiana, north dAKota, ALaska, alabaMA, Massachusetts.