SYMMETRIC CRASH GROUPS

A. ROSS ECKLER
Morristown, New Jersey

Two words of the same length are said to crash if they have matching letters in one or more positions, such as tiger and pious, or round and attend. A symmetric crash group is a set of words in which (1) each word crashes exactly \( n \) times with each other word, (2) every letter participates in a crash, and (3) each letter is used exactly \( m \) times in a given position. Perhaps the simplest illustration of such a group is given at the right: four three-letter words crash each other exactly once (PEN and SET on E, PEN and POT on P, etc.). Each letter is used exactly twice, and each letter participates in one of the crashes.

Can one construct larger and more elaborate crash groups? The group illustrated above is the simplest member of a family of potential symmetric crash groups of \( 2^i \) words of length \( 2^i - 1 \) \((i = 2, 3, 4, \ldots)\). The next member of the family, consisting of six five-letter words, each crashing once with the others, is given at the right. This example is unusual in that all the words can be found in boldface type in the Merriam-Webster Pocket Dictionary, or inferred from such boldface words; furthermore, fifteen different letters are used (no letter repeats in a different position). Can anyone find another example having these characteristics (other than the trivial substitution of M for W)?

The next member of the family, a group of eight seven-letter words each crashing once, is considerably harder to find. It may be possible to locate a group of words from Webster's New International Dictionary, Second or Third Edition, but it seems far more likely that words outside these references will have to be used. For the record, two near-miss Websterian groups are given at the right. The lower-case letters are the only ones to fail the crash pattern; if N, D, L replaced p, c, e, and N, E, O, C respectively, then both groups would be perfect. (It is obviously impossible to insist that each pair of letters be unique, for there are 28 such pairs, two more than the number of letters in the alphabet.)
As suggested by the similarity of the two examples, the search for a perfect group is best begun by assembling a set of common three-letter endings with a proper crash pattern (these are readily identified with the aid of a reverse-English dictionary). To make the search more systematic, it is worth copying down all seven-letter words with each three-letter ending (on eight sheets of paper) using the Air Force Reverse English Dictionary of Webster's Second to get as large a stock of words as possible. It is best to try and fit in first words with the rarest endings, or words in which the choice of earlier letters is particularly restricted. For example, of the 23 seven-letter words ending in OGY, 22 end in LOGY and 18 in OLOGY; these two letters must almost certainly be incorporated in any solution.

In selecting common three-letter endings, one must balance common ones against rare ones that are linked with them. For example, the ending -ING should probably be avoided, for all matching endings of the form -G are relatively uncommon.

It is worth noting that these symmetric crash groups are closely related to certain word groups described in the May 1977 Word Ways -- in particular, to the partially overlapping word groups CAN, COT, ATE, ONE and WRIST, WHOLE, COUNT, CHARM, PLAIN, SPUME in which each word has one letter in common with each other word, and each letter appears in exactly two words. To make the correspondence more striking, the letters of the words in the second group can be rearranged to form the earlier letter pattern of HATED, HORNY, FITLY, FAUNS, WIRED and WOULD. (Since no letter could appear more than twice in the partially overlapping word groups, there is no analogue to the symmetric crash group of eight seven-letter words.)

So far, we have considered only those symmetric crash groups in which each word crashes once with each other word, and each letter is used twice in a given position. Both of these conditions can be altered; for example, at the right is a group of four six-letter words with three mutual crashes. Similarly, one can use each letter three times instead of twice, as the symmetric crash group at the right demonstrates; note that each word crashes exactly once with the others. This can be generalized to a group of sixteen five-letter words, with each letter repeated four times in a given position, again with each word singly-crashing the others. It should not be too difficult to construct an example.