PLATONIC RELATIONSHIPS

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In the November 1978 Kickshaws, Harry and Mary Hazard presented a word game in which two players alternately label with letters the vertices of a cube, each player's object being to form words encircling the square faces and block the other player from doing this. The game suggested the following challenge: label the vertices of each of the five Platonic solids (tetrahedron, cube, octahedron, dodecahedron, icosahedron) with letters of the alphabet, none repeated, so that words can be formed out of the letters assigned to the vertices of each face.

When the faces are triangular (as in the tetrahedron, octahedron and icosahedron), it is always possible to read off the words by proceeding sequentially around the vertices of each face. However, when the faces are square (as in the cube) or pentagonal (as in the dodecahedron), rearrangement of the letters may be necessary. For the cube, the additional restriction can be imposed that the letters form a word when read off in sequence, but for the dodecahedron this proved to be impossible.

The challenge can be reformulated by writing the letters on the faces rather than the vertices of the Platonic solids, and reading off the words formed by those faces having a common vertex. In this case, the dodecahedron and icosahedron exchange roles, as do the cube and octahedron; however, the tetrahedral problem remains the same whether faces or vertices are labeled.

The groups of words formed by Platonic inscription are closely related to the balanced word groups described in detail in the May 1977 Word Ways. Both the Platonic word groups and the balanced word groups consist of $m$ different letters, each used $n$ times to form $k$ words of $r$ letters each ($mn = rk$). However, the Platonic word groups in general lack two further balance-properties: each word overlaps each other word in the same number of letters, or each pair of letters appears together in the same number of words. In the case of the tetrahedron, however, the two word groups are identical -- a tetrahedral Platonic group is precisely a Baltimore transdeletion of three-letter words, such as SET, SEA, SAT, EAT. (A Baltimore transdeletion of a set of $n$ letters is formed by successively deleting each letter and forming words out of the remaining $n - 1$.)

The next easiest Platonic word group consists of eight three-letter words formed out of six letters assigned to the vertices of an octahed-
ron. There are many ways to solve this; I suggest NOR, PRO, POT, NOT, RAN, PAR, PAT and ANT, as illustrated in the diagram at the right.

The cubic Platonic word group consists of six four-letter words formed out of eight letters assigned to the vertices. As noted earlier, it is not necessary to rearrange the letters to read off the words OVAL, DIRE, DOVE, LIRA, RAVE and IDOL.

Next in difficulty is the icosahedron, to which one assigns twelve different letters and obtains twenty three-letter words. It does not seem to be possible to restrict all twenty words to boldface entries (no abbreviations) in the Merriam-Webster Pocket Dictionary, although there are a number of solutions in which nineteen appear. We present one in which the twentieth word is a common male nickname (as well as a word in Webster's Unabridged): ACO, ADO, AID, AIR, RAG, HOT, DOT, TED, DIE, PIE, FIR, FUR, RUG, HUG, HOG, NTH, HUN, FUN, FEN, NET. The diagram at the right is presented in distorted (flattened) form; the isolated letter, A, is connected to D, I, R, G and O. Readers who tackle the problem of trying to find twenty Pocket Dictionary words are warned of inherent constraints: one must either use all six vowels AEIOUY or an all-consonant word (NTH is the only one in the Pocket Dictionary). Further, the six vowels can be arranged in either an open-ended string in the diagram above, or else in a closed ring of five with one isolated vowel.

The hardest Platonic solid to label is undoubtedly the dodecahedron: twenty different letters of the alphabet form twelve five-letter words. A Pocket Dictionary solution is impossible, and even one using boldface entries from Webster's Second and Third editions is not easy (I was materially aided by a transpositional dictionary of five-letter words from Webster's Second, compiled by Tom Kurtz of Dartmouth College). The set of words is CHOWK, BUMPS, CHIMP, RHOMB, SPILD, RUGBY, GENTRY, GROWN, WAKEN, STUDY, ALICK and DELTA. In the diagram, the twelfth pentagon is identified by the five outermost vertices.