SINGLE AND DOUBLE TRANSPOSA L SQUARES

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A transposal square is a square array of letters in which each row and column can be rearranged to form a word. Philip Cohen introduced the concept to Word Ways readers in his November 1979 Kickshaws, presenting there a 10-by-10 double transposal square (ten distinct letter-combinations arrangeable into words in the rows, and ten additional distinct letter-combinations arrangeable into words in the columns). This article exhibits a 12-by-12 double transposal square produced by his construction method, as well as an 11-by-11 single transposal square (eleven distinct letter-combinations arrangeable into words in the rows, and the same eleven letter-combinations in the columns).

No general method for constructing transposal squares is known. The method presented in Kickshaws results in certain highly symmetric transposal squares, probably a very small fraction of the total possible. Should more general methods be discovered, or should a computer be assigned to the task (even on a trial-and-error basis), it is likely that larger squares of both varieties could be found.

The Kickshaws method is based on the mathematical concept of a Latin square -- an n-by-n square containing n copies of n different letters, arranged so that each letter appears exactly once in each row and column. (These squares are used in the design of statistical experiments, with the letters representing competing varieties of treatments or products under test, and the rows and columns different test conditions.) To construct a single transposal square, one replaces a single letter in the Latin square with n different letters, thus ensuring that each column has a unique set of letters. By proper selection of the common base, as well as the replacement letters, one can rearrange each set of letters to a word.

To fix ideas, consider the common base of letters ACEILNORTU. These ten letters are placed in the 11-by-11 Latin square on the next page; the eleventh letter of the Latin square, along the main diagonal running from upper left to lower right, is replaced with the letter-set CDEGKNORSTV. The eleven Websterian words produced by this selection are given at the right of the square; because of the symmetry, the same words are reproduced in the columns. Unlike regular word squares, any pair of rows or any pair of columns can be interchanged without destroying the Latin square property and the row and column words; the square exhibited here has a pleasing visual pattern.

It is conjectured that the 11-by-11 square shown here is the largest possible single word square from the bridged dictionary (not all have sufficiently adapted n-letter words; more tranposal squares out of its n-letter set appear in the May 1977 different Webster and ACEILNORTU; chlorine, clarion, colo, clarion found ACEILNORTU proposals supply choice of letters not enough.)

A double transposal square, same ideas: an n-by-n square containing n distinct one to n different letters. (Again such a square is conjectured here to be the last such a square of this kind). Again, by a selection of a common base and replacement letters, one can rearrange each set of letters to a word.
Each row and column is known. The total possible. A double transposal square can be constructed by an extension of the same ideas: instead of replacing a single letter in the Latin square with n distinct ones, one must replace two letters in the Latin square with 2n in such a way that no row or column contains the same two letters. (Before such a substitution, all rows and columns have the same two letters.) Again, an example:

<table>
<thead>
<tr>
<th>Row words</th>
<th>Column words</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACEILNORTU</td>
<td>corniculate</td>
</tr>
<tr>
<td>UDACEILNORT</td>
<td>radiolucent</td>
</tr>
<tr>
<td>TUEDACEILNOR</td>
<td>reinoculate</td>
</tr>
<tr>
<td>RTUGACEILNO</td>
<td>out-clearing</td>
</tr>
<tr>
<td>ORTUKACEILN</td>
<td>unactorlike</td>
</tr>
<tr>
<td>NORTUANACEIL</td>
<td>crenulation</td>
</tr>
<tr>
<td>LGINORTUACEI</td>
<td>unicolorate</td>
</tr>
<tr>
<td>EILNORTUSAC</td>
<td>interocular</td>
</tr>
<tr>
<td>CIELNORTUTA</td>
<td>ulcerations</td>
</tr>
<tr>
<td>ACEILNORTUV</td>
<td>countertail</td>
</tr>
<tr>
<td>CORNICTULATE</td>
<td>facevail</td>
</tr>
</tbody>
</table>
| RADIOLUCENT | }

The problem is to find a set of (n - 1) base letters (not all have to be different, although they were in this example) sufficiently adaptable that n different letters can be transadded to form n-letter words. (The problem is inverse to the one of forming a Baltimore transdeletion, in which one takes a set of n letters and forms words out of its n different subsets of (n - 1) letters. In Webster, the largest set appears to be ACEILNORST, as demonstrated in "Word Groups" in the May 1977 Word Ways.) Guided by the fact that ACEILNOR forms 17 different Websterian words (see "Caroline" in the May 1971 Word Ways) and ACEILNORT forms 15 (laceration, centroidal, francolite, relocating, chlorinate, citronella, centimolar, interconal, relocation, goniath, ulceration, intervocal, intercoxal, lectionary), I found ACEILNORTU by trial and error from a list of Websterian transposals supplied me by Tom Kurtz of Dartmouth College. (The next best choice of letters is ACEEILNORT, which has only ten transadditions -- not enough.)
In the 12-by-12 double transposal square on the previous page, the base letters are AACEILNORT, and the 24 substituted letters lie along the main diagonal, the diagonal immediately above it, and the lower left-hand corner. This example is not quite perfect, for one two-word dictionary phrase, GOLIATH CRANE, had to be used. This defect can in principle be corrected at the cost of destroying the symmetry; all that needs to be done is change any letter (other than G or H) in the eleventh row to produce a solid word, making sure that the letters in the correspondingly-altered column also form a word. Alas, GOLIATH CRANE does not appear to have a Websterian substitute-letter transposal!

It is not an easy task to construct a 12-by-12 double transposal square. One must identify a set of ten letters which produce somewhat more than 24 double transadditions, and one must then arrange 24 of these double transadditions in a closed ring. For example, the letter set AACEILNORT generates words with the following 32 transadditions (asterisks denote two-word phrases): ad, au, be, bn, ce, ch, cm, cv, di, ds, er, fs, gh, gi, gm, go, gp, gv, hp, 10, lp, lt, ly, ms, nx, or, ps, pu, st, su* -- probably the largest number of Websterian twelve-letter words having a common base of ten letters. If one examines the pattern of the 24 substituted letters in the double transposal square, one can see that these are arranged in a closed ring: starting at the upper left sp, pl, io, og, gi, id, ... down to hg, gb and then back to the start via the lower left corner with bn and ns. This ring can be formed as follows: (1) construct a network of twenty-one letters abcdefghilmnopstuvxy, joining all those letters in the list of 32 letter-pairs above with links; (2) by trial and error, locate a closed ring of 24 links (visiting letters more than once if necessary, but never passing along the same link twice).

It is conjectured that the 12-by-12 square is the largest possible double transposal square based on a Latin square using only Websterian words. It is unlikely that any set of eleven letters will produce as many as 26 double transadditions -- and, in fact, more than 26 are needed, for a closed ring enter and leaves each letter an even number of times, wasting a link every time 1, 3, 5, ... links emanate from a letter.

BUY, SELL, TRADE

Language on Vacation and Beyond Language, offered used in the February 1980 Word Ways, have both been sold. As there were several inquiries about the latter book, it may pay Word Ways readers to watch for this title in old book stores, offering it for resale in this column.

FRENCH

LEONARD

Brooklyn, N.Y.

I hate to du sucire (do sugar literally),

Some are to help.

Try to meet too much too.

1. That will

2. Don't try

3. You run

4. To hand

5. To give

6. To make

7. ... and

8. In two thes

9. I see you

10. Ingres'

11. He's us

12. Admire

13. Drink li

14. To roll li

15. He paid

16. In the m

17. Like a D

18. He break

19. Work for

20. I'm not

21. To go li

22. To put it

23. Like a cu

24. Snuff hi

25. I've lost