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In the November 1978 Word Ways, Mary J. Hazard introduced a word game by Dave Silverman in which two players alternately assign letters to the vertices of a cube with the objective of forming four-letter words on the vertices of each face. Specifically, a player loses the game if he is unable to form words on all faces on which his letter fills in the last open vertex. If this never occurs, then the player who fills in the last open vertex on the cube wins the game.

It is easier to write on the faces of a Platonic solid than on the vertices. Words formed on the faces meeting at a common vertex.

If an octahedron is substituted for a cube, Dave's game remains the same.

It is in principle possible to set down a strategy for this game, exhibiting those letters (if any) that ensure a win for the first player, together with a tree of actions he must take for every possible second player move. To give the flavor of this strategy, I analyze a much simpler game -- one played with three-letter words on the four faces of a tetrahedron, restricted to words found in boldface in the Merriam-Webster Pocket Dictionary (1970 edition).

There is no three-letter word containing Q in the Pocket Dictionary. To avoid reducing the game to triviality, I propose that Q not be admissible as a letter. If Q is allowed, the second player can always win by selecting it; the first player is unable to make a further move and loses.

I propose also that repeated letters be prohibited. This simplifies the analysis slightly, and makes the game more interesting by assuring that four different words are formed when the faces are all assigned letters. In fact, these words form a Baltimore transdeletion (select n different letters and form words out of all possible subsets of n - 1 letters).

The tetrahedron game, like tic-tac-toe, is uninteresting to play once the strategy is known to both players, for the first player can guarantee a win no matter what the second player does. The interest lies solely in the discovery of the optimal strategy by players learning the game through experience. (For games on most other Platonic solids, this learning will take a very long time, and it is unlikely that the optimum strategy will ever be known in full.)
Briefly, the first player can ensure a win by playing the letter A and responding to his opponent's move with the paired letter below:

B, T  F, G  J, R  M, P  S, W
C, E  H, S  K, U  N, Y  O, K

For example, if the opponent selects T, the first player forms BAT with B, and there is no letter that can be used by the opponent that forms a word when added to A, B, to A, T and to B, T (unless a larger dictionary is allowed).

Why are the other letters unsafe for the first player? The only Pocket Webster isograms with Z are ADZ, FEZ, ZED, ZEN and ZIP so all but seven letters can be countered by Z by the second player.

To block D, F, P and N, he need only use J instead; to block I, he can use U.

The strategy for E is a bit more interesting. Here, the second player must play the common letter O, forcing the first one to use D, F, G, H, L, N, P, R, T or W to form a word. For each of these, it is possible for the second player to win by completing a Baltimore transdeletion, as shown below:

EOD  ode, ore, rod, red       EON  one, toe, ton, ten
ECF  foe, oft, eft, toe      EOF  ope, hoe, hop
EGG  ego, log, leg, Leo     EOR  ore, ode, rod, red
EHO  hoe, who, woe, hew     EOT  toe, foe, oft, eft
EOL  Leo, log, leg, ego

If an expert challenges a neophyte to the tetrahedron game, he can conceal the optimal strategy for awhile by mixing it with strategies that are only slightly unsafe. For example, he can offer to let the neophyte go first, taking a chance that he will not select A. If he does select A, the best strategy is to select the second letter E or O. If the neophyte selects a letter at random to complete the word, the chances are overwhelming that the expert can complete a Baltimore transdeletion and win:

AEC  no transdeletion       AET  tea, set, sat, sea
AEG  age, get, tag, ate     AEV  ave, vet, vat, ate
AEL  lea, lye, aye, lay     AEW  awe, wet, taw, ate
AEP  pea, pet, pat, tea     AEX  no transdeletion
AER  are, rye, ray, aye     AYE  aye, rye, ray, are
AES  sea, set, sat, tea

AOB  boa, bog, bag, ago     AOK  no transdeletion
AOD  ado, gad, god, ago     AOR  oar, rot, rat, oat
AOF  oaf, fog, fag, ago     AOT  oat, rot, rat, oat
AOG  ago, got, tag, oat

If the neophyte selects I, the expert can avoid revealing the sure strategy of U (no word containing U and I, as already noted) by picking E;
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again, his chances of completing a Baltimore transdeletion are high:

| IED die, red, rid, ire | IEP ple, hep, hip, hie |
| IEF lie, eft, fit, tie | IER ire, red, rid, die |
| IEH hie, hip, hep, ple | IET tie, let, lit, lie |
| IEL lie, let, lit, tie | IEV no transdeletion |

Sooner or later, the neophyte will tire of always losing and propose
that the experienced player go first. To conceal the A-strategy as long
as possible, the expert should select I, E, O and Y as well; the chances
of a win for him are very high unless the neophyte is lucky. For an in-
itial choice of E, the expert should respond to his opponent's move
with the paired letter below:

| A, C | F, Z | I, V | P, N |
| B, Y | G, K | J, T | S, X |
| D, U | H, R | L, M | W, B |

As already noted, the expert loses if the opponent selects O.

For an initial choice of I, the expert's responses are:

| A, M | E, X | H, S | N, X | T, W |
| B, D | F, R | K, P | O, N | V, Y |
| C, Y | G, J | L, M | P, Z |

As already noted, if the opponent selects U the expert loses at once.

For an initial choice of O, the expert's responses are

| A, K | D, U | I, N | S, X |
| B, R | F, G | J, T | V, W |
| C, P | H, L | M, W | Y, B |

As already noted, if the opponent selects Z the expert loses at once,
and if he selects E the expert loses on a Baltimore transdeletion.

For an initial choice of Y, the expert's responses are:

| A, K | D, R | H, T | L, S |
| B, U | E, F | I, V | N, A |

As already noted, if the opponent selects Z, the expert loses at once.

W is also unsafe, as the table below indicates:

| YWA way, awe, aye, yew | YWH why, haw, hay, way |
| YWE yew, aye, awe, way | YWR wry, raw, ray, way |

The sixth vowel, U, is a bit more risky to use as there are five let-
ters which fail to form words with it: I, V, W and Z.

What is the optimal strategy for a game played on the faces of a cube?