5X5X5 WORD CUBES BY COMPUTER

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Recently, I used a computer to look for 5x5x5 word cubes: symmetric patterns that spell five-letter words in rows, columns and "whatchamacallits" (is there a name for vertical 1x1x5s?). As far as I know, the only previous example was constructed by Peter Graham and published in Omni in July 1987. But his solution:

LOVED OPE R A VEL AR ERA SE DARES
OPE R A PUR E R ERO DE R E D A N A RE N A
VEL AR ERO DE LOGOS ADO RE RESET
ER AE S R E D A N A D O RE S A R U M E N E M Y
DARES A RE NA RE SE T E NE M Y S A T Y R

uses REDAN and SARUM, which are not in my vocabulary. Also VELAR is kind of hazy for me. I know it's a word, but I don't think I've seen it in novels or "real" English.

Lots of obscure five-letter combinations can be found in unabridged dictionaries, but word puzzles are much more fun when they stick to words that I wouldn't feel like challenging if my opponent played them in Scrabble. So I've been collecting a list of all five-letter words that I personally know and love, during odd moments since the early 1970s. Last week I decided to see if any 5x5x5 cubes were possible using only words from my collection (5757 in all) ... and I found to my surprise that there are many, many solutions (exactly 83576)!

Here are the five best, as far as I can see:

ASTER SCALE TACOS ELOPE RESET
SCALE CODEX ADAPT LEPE R EX TRA
TACOS ADAPT CAPER OPERA STRAP
ELOPE LEPE R OPER A PER K S ERA SE
RESET EXTRA STRAP ERASE TAPER

TYPES YEAST PASTA ESTER SE R E T
YE AST EARTH ARM OR STOLE THREE
PASTA ARMOR SMOKE TOKEN ARENA
ESTER STOLE TOKEN ELECT RENTS
START THREE ARENA RENTS TEASE

ABLER BLADE LAP IS EDICT REST S
BLADE LOG IN AGENT DINAR ENTRY
LAP IS AGENT PESTO INTRO STOOL
EDICT D INAR INTRO CARGO TROOP
REST S ENTRY STOOL TROOP SYLPH
Each of these has one somewhat unusual word: CODEX, ESTER, LOGIN, MOPER, TALCS. But they all pass the Scrabble test, except that LOGIN might not yet be in standard dictionaries ("What is your login name?").

Another solution uses no unusual words whatever, but it has a flaw in that it uses ERASE twice (or more times, depending on how you count):

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RACER  ADORE  COMAS  ERASE  RESET
ADORE  DIVAN  OVERT  RARER  ENTRY
COMAS  OVERT  METER  ARENA  STRAP
ERASE  RARER  ARENA  SENDS  ERASE
RESET  ENTRY  STRAP  ERASE  TYPE
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Notice that the final square here is the same as in the fourth solution above.

The vast majority of the 83576 solutions contain too many rare words to be really interesting.

Why are there so many solutions? Because each solution can usually be tweaked into another by changing only a few letters. For example, you can easily find 20 different solutions that fill in the *s of

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A*ER  ADORE  *OAS  ERASE  RESET
ADORE  DIVAN  OVERT  RARER  ENTRY
*OAS  OVERT  *EER  ARENA  STRAP
ERASE  RARER  ARENA  SENDS  ERASE
RESET  ENTRY  STRAP  ERASE  TYPE
```

and 57 ways to complete the pattern

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S*ER  SCALE  *AOS  ELOPE  RESET
SCALE  CODEX  AD*PT  LEPER  EXTRA
*AOS  AD*PT  L**E  OPERAST*A
ELOPE  LEPER  OPERATE  PERKS  ERASE
RESET  EXTRAST*A*ERASETA*E
```

Here is a list of common words to fill in the asterisked patterns:

*OAS (sodas, sofas, novas, comas, yogas, togas, iotas, colas, codas, bolas, cocos, somas, soyas, molas); *ASER (laser, baser, maser); *ANEER (saner, caner); *ACER (racer, pacer, facer, macer, lacer); *AYER (layer, gayer, payer, sayer, hayer); *ATER (water, later, caner, refer, me
later, cater, eater, mater, hater, pater, rater, tater, dater); AbE (saber); AbE (tamer, lamER, hamer, gamer); e*ER (never, refer, meter, fewer, fever, lever, newer, sewer, deter, sever, leper, defer, peter, sedER, weVer, weber, ceder, hexer, keyer); Tye* (types, typed). I have ranked the words in my corpus by frequency of use in various texts; the common words listed above have ranks of 3500 or less. If the entire list of 5757 words is used, I get not 20 but 110 different solutions. The point here is not to obtain an exact count, but rather to illustrate the multiplicative phenomenon that accounts for the large number of word cube solutions.

If my stockpile of five-letter words is restricted to the most common 3000, there are no word cubes possible. Expanding the list to 3500 results in 83 solutions, including the one in the middle of the preceding page.

but it has depending on EX, ESTER, test, except ("What is the fourth many rare solution can few letters. s that fill PR E TRY T E R A P A T E S T E E P S T E P S ST E P S S T E P S

GRASS GLASS CLASS
RIGHT LIGHT LIGHT
AGREE AGREE AGREE
SHEEP SHEEP SHEEP
STEPS STEPS STEPS

No further solutions appear until the 711th word, ALARM, which yields the square

PARTS
ALAR
T R I C K
S M O KE

patterns:
at, colas, ser, baser, cer, macer, TER (water,