AN ABELIAN ALPHABET?

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A curious problem may be stated very simply for those familiar with the language of mathematics. Consider the free group on the 26 letters of the alphabet, with transposals as relations; is the group Abelian?

What this amounts to for the non-mathematician is the following. Words are considered as strings of letters, as is so often the custom in Word Ways, and we say two words are equivalent if they are transposals of each other. For example, EAR=ERA, RACE=ACRE, and UNITE=UNTIE. In such an equation, or relation, between two words, we are allowed a simple cancelling process. If both words have the same initial letter, or if both words have the same final letter, we can cancel it on both sides. For example, EAR=ERA results in AR=RA. RACE=ACRE in RAC=ACR, and UNITE=UNTIE in NITE=NTIE.

The result is obviously an equation between strings of letters only, since in general the cancelling process rarely results in words. Applying the cancelling process twice more to NITE=NTIE results in IT=TI. The ultimate goal is to provide sufficiently many transposal pairs to conclude that xy=yx, where x and y represent any two distinct letters of the alphabet. For example, we already have RA=AR and IT=TI from the examples above. In the jargon of mathematics, xy=yx is described by saying that x and y commute.

It is reasonably clear that it will prove well-nigh impossible to show, for instance that QZ=ZQ by the same technique as that of the example UNITE=UNTIE, since this would necessitate finding two transposals, one containing the digram QZ, the other ZQ, as well as both words containing the same cancellable letters. This difficulty is overcome by simplifying an equation using any equations between letter strings that have already been derived. For example, we have seen that RACE=ACRE results in RAC=ACR, but we also know that AR=RA, and it follows that ARC=ACR, so that ARC=RAC=ACR. From ARC=ACR now follows RC=CR on cancelling the initial A. Another example: CARE=RACE, so CAR=RACT (cancellation of final E), so CRA=RAC (using AR=RA), so RCA=RAC (using CR=RC), so CA=AC (cancellation of initial R). It is essential when cancelling that it be performed only on the leftmost, or rightmost, letters. From CAR=RAC it is impossible to deduce immediately that CA=AC; the auxiliary information given by RA=AR, RC=CR is needed.

Once the process has been mastered, it is quite easy to reduce an equation between two transposals. It is not too difficult, and a good exercise, to show first of all that each of the vowels AEIOU commutes with every other letter of the alphabet. This then means that in any future transposal pair, the vowels can be ig-
To show that $x$ and $y$ commute, a transposal pair must be found in one word of which $x$ precedes $y$, and in the other, $y$ precedes $x$. This is where the difficulty begins, because of the intractable letters $J, Q, X, Z$. If the whole problem is to be solved, then it seems that recourse to large dictionaries must be allowed. On aesthetic grounds, however, hyphenated words (e.g. SIXTY-SEVEN=SEVENTY-SIX) should be avoided. On a personal whim, I have also excluded two-letter words.

In practice, I have found it best to draw a $26 \times 26$ grid, filling in the $xy$ square (and the $yx$ square) when $x$ and $y$ have been shown to commute. This allows seeing immediately the letter pairs that already commute, which helps in writing down suitable further transposal pairs. The following transposals have been taken from Chambers Twentieth Century Dictionary, the Merriam-Webster Unabridged dictionaries (Second and Third Editions), Funk & Wagnalls Unabridged, and the Oxford English Dictionary (asterisked words are under a different head-word therein). The list shows that every possible pair of letters commutes, with the exception of $\text{JX}, \text{KQ}, \text{QW}, \text{QX}, \text{VX}$ and $\text{XZ}$. The listing of bigrams follows a specific nonalphabetic order: each letter pair can be shown to commute using transposals earlier in the table. The choice of transposals is, of course, arbitrary; there are many other possibilities for the common letters.

To complete the problem of showing that the alphabet is Abelian, it is necessary to find transposal pairs for the six missing cases. I look forward to seeing in Colloquy examples found by readers!

There are several variations on the original problem. For example, within the confines of a relatively small dictionary, is it possible to show that just the vowels AEIOU commute with every other letter?

AB abba=baba AC act=cat AD add=dad AE tae=tea AF aft=fat AG agin=gain AH tahr=char AI ain=ain AJ ajaw=jawain AK akin=kain AL alp=lap AM amp=map AN any=nay AO goa=goal AP apt=pat AR ear=era AS asp=asp AT eat=eta AU gaur=guar AQ aqua=quaa AV ave=vae AM awe=wae AX coax=coxa AY aye=yaZ azo=zaB BC bac=cab BD bad=dab BE bee=ebB BG bag=gab BK bak=kbB BL bal=lab BM baml=mbB BN bnn=bob BO boe=obe BF bpf=pbF BR bar=rbB BS bas=sbB BT bat=tab BW ba#w=wbC CD cade::dace CE cetene=etene CF cafe=face CJ caja=jaca CL cal=lac CM cam=mac CN cane=nace CP cap=pac CR cark=rack CS cask=sack CT cate=tace CU caul=ucal CV cavate=vacate CW cawk=wack DE deify::edify DG dag=gad DH dah=dah DI die=die DK daku=kuD DL dal=lad DM dam=mad DN dander=nadder DO ado=oda DP dap=pad DR dar=dad DS das=sad DT date=tade DU davo=vader DW dew=daw DY day=yad EF lif=life EG egal=geal EH seah=seah EI lel=lie EI eel=keE EL elt=let EM emane=meane EN eam=nae EO leo=loe EP reap=ep RE era=era ES ese=ese ET ate=eat EU leu=leu EV eever EW eee=eeX EX exam=xema EY eyed=yeed EZ ezel=zeel FO fot=oft FR fort=raft FY fay=yaf GJ gaj=gaj

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GL gal=lag GM gam=mag GN gan=nag GO goam=ogam GP gape=page GR gar=rag
GS gas=sag GT gat=tag GU gae=vage GW gaw=vag GZ gaz=zag
HJ haj=jah HK haka=kaha HL hal=lah HM ham=mah HN han=mab HO pho=poh
HP hap=rah HS ash=has HU hullo=uhlo HW haw=vah HY hay=yah
IL ilka=laik IM imam=maim IN ins=nis IO lion=loin IR air=ria IS isle=isle
JR jar=raj JT jat=raj KL kale=lake KM kame=make KO koa=ook KR kari=raki
KT kat=take KU kau=vuk KV kavass=vakass KW kawa=waka KY kay=yak
LM lam=mal LN lana=nala LO lope=olpe LP lap=pal LT late=tale
LU glue=vale LX axl=lax LY yal MN man=ram MO mit=omit
MP map=map MR mar=ram MS mas=sam MT mat=mat MU muset=umset MW maw=vame
MY may=yam NZ maza=zama NO nous=onus NP nap=pan NR nar=ran NS nas=san
NT nat=nan NU nuan NV nave=vame NW wan=wam NY nan=yan OP opt=opt
OR ort=rot OS ose=ose OT oter=toter OU mou=mou OV ovile=ville
OW owe=owe OY owe=owe OZ ozonic=zoonic JP japone=pajone PR par=par
PS pas=spa PT pat=pat PU pus=pus PV paw=paw PY yap=yap RS ras=ras
RT rat=rar RU rue=rue RV rave=rave RW raw=rar RY yar=yar ST star=star
SU sue=use SV save=vave SW saw=saw SY say=sas TU tau=tau TW taw=taw
TY tay=yat UV vau=vau WY way=yaw IT ite=time BI bit=tib CI cima=mica
FI fire=ripe GI git=rig HT het=the HI whit=with IK kite=ike IP hip=hip
IV live=lire LR leer=leer LS lose=sole KN kem=nek CH chi=chic
CK hackin=kachin FK fakir=kafir GC gie=rice CO canoe=ocean
BF brief=fibre BU bun=mub BN bush=bubs BW observe=observe BY yerra=bary
DF deaf=fade FN fen=feef FS fist= Fist FL file=life FH flesh=shelf
FT feathers=tashref LW lewd=veld FW flow=lof PG flog=golf FU furs=surf
FM flemish=felsh GK gink=king GR charge=creagh
UY yupa=yuyu CY cuya=gyu CV gue=vace UY viability=vyngvi
IU inured=ruined IV inew=wise NZ nazi=zaiz IN niz=ziin KP keep=keep
KS inks=skin JS jambos=jamb MK jamas=mom MV mover=move
LZ laze=zeal LW tove=vote UW unware=wauner VW vows=vovles JO joyo=soja
BJ bojo=jobo DJ dojo=dojo JK jaw=kawu EU join=join UI uis=ius
JQ jaques=quaas JQ geom=geem JL jena=jaen JL jela=jell PJ jiff=fijf
JV javar=vaar JW jewis=wiljes* JY joyo=yojo JZ jiz=jizz FV fveye=fve
IX myri=yrn PV passover=passover QS quads=quads EQ equals=equll
QR quest=rquest EQ equals=asque IQquires=risque MQ masque=squame
QU qu=soqu EQ equip=equipe QT quality=tequila QU quote=quot
FQ fique=queef BQ base=basque RQ requies=risesque MQ massque=squame
HQ cheque=queeh DQ dequeen=ened QG gasque=queasing
QY quyty=yqueyt RZ razed=zerda BR braze=zebra DZ adzed=zedad
HZ hazel=zelaz CZ close=zoce UZ chuize=zuize QZ quize=quequin PZ fiz=zf
PR phi=zip WZ wiz=ziw SZ swet=zeus TZ nietzsche=zeitschein
HX exhale=hallex DX detox=taxed RX retax=taxer UX exul=ulex
SX nexus=unsex BX exalt=latex IX sixty=ysty XY styst=cyx CX cylix=xyl
NX unwaax=xwaun* OX axon=axoa WX xowyn* WQ xowyn BX boxer=xorbr
FX foxier=xeffer* YZ phytozoa=zoophyta KZ kozos=zkoks PX expos=poses
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VZ veze=zeve

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