WORD LADDER SQUARES AND CUBES

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In its traditional form, the word ladder is a one-dimensional structure; even word ladder networks with many branchings proceed in a basically linear fashion. But one can conceive of word ladders which extend solidly in two and three dimensions, in the manner of the familiar word square and word cube. This article will suggest some definitions and rules of construction which might reasonably apply to such arrays, and offer some illustrative examples.

DEFINITIONS

If the two end words of a word ladder have no letters in common at the same positions, and if the ladder contains only the minimum number of steps needed to completely transform words of that length, the ladder is known as an ideal word ladder (example TIME-TIRE-WIRE-WARE-WARP). Ideal word ladders are the basis for the classification scheme which follows.

If in a square array of words ("square" in the sense that its rows and its columns have the same number of words) every row and every column is a word ladder, the array is a word ladder square (WLS). The three-dimensional counterpart of the WLS is the word ladder cube (WLC). Other two- and three-dimensional word ladder shapes may exist, of course, but these will not be considered here. Also excluded from consideration will be squares or cubes in which word ladders may be deemed to be anything other than single rows or columns. The suggested minimum "regulation" size for a WLS is a square of side four, and for a WLC it is a cube of edge four. Further definitions:

Simple WLS or WLC: one in which there are no ideal word ladders

Ideal WLS: one in which at least one, but not all, of the word ladders is an ideal word ladder

Perfect WLS: one in which every word ladder is an ideal word ladder

Ideal WLC: one in which at least one, but not all, of the constituent WLSs is ideal

Perfect WLC: one in which at least one, but not all, of the constituent WLSs is perfect
Sublime WLC: one in which all of the constituent WLSs are perfect

In addition, it is important to distinguish between what might be called proper and improper word ladder arrays. The latter are essentially trivial arrays which are easily produced by the systematic repetition of segments of word ladders; as such, they cannot be considered logologically challenging, although they do possess a number of points of interest. The rule which distinguishes proper from improper arrays will be described in the next section.

Heterogeneous word ladder arrays are those in which all of the words are different. Proper WLSs and WLCs may be either heterogeneous or nonheterogeneous.

Lastly, a standard word ladder array is one which contains no ladders in which the same word appears more than once, and a nonstandard array is one which contains at least one such ladder. Except where otherwise noted, the word ladder arrays discussed in this article are all standard arrays.

PROPER WORD LADDER SQUARES AND CUBES

The Two-Word Sequence Rule  A given word may appear more than once in both proper and improper WLSs, but in a proper WLS the same two-word sequence (not combination) may not appear more than once in the rows and once in the columns. For example, in a proper WLS the sequence TALL-TALE, if it appears in a row, may not appear in any other row, but it may appear in one column; the sequence TALE-TALL, however, being considered a different sequence, may appear once in the rows and once in the columns of the same square. In a proper WLC, this rule applies to the individual constituent WLSs but not to the WLC as a whole. The reason for choosing this particular rule (which I call the "two-word sequence rule") to distinguish between proper and improper WLSs and WLCs will be discussed in the section on improper word ladder squares and cubes.

Three-Letter Words  Ideal and perfect WLSs of three-letter words are easily found. Of the four which follow, the first is ideal and the rest are perfect:

<table>
<thead>
<tr>
<th>CAT</th>
<th>COT</th>
<th>COW</th>
<th>DOG</th>
<th>PAN</th>
<th>PEN</th>
<th>PET</th>
<th>RAT</th>
<th>BAT</th>
<th>BOT</th>
<th>BOG</th>
<th>RIP</th>
<th>REP</th>
<th>REX</th>
<th>VEX</th>
</tr>
</thead>
<tbody>
<tr>
<td>CAD</td>
<td>COD</td>
<td>COT</td>
<td>DOT</td>
<td>FIN</td>
<td>PIN</td>
<td>PUN</td>
<td>PUT</td>
<td>RET</td>
<td>BET</td>
<td>BIT</td>
<td>BIG</td>
<td>RID</td>
<td>RED</td>
<td>RET</td>
</tr>
<tr>
<td>COD</td>
<td>mod</td>
<td>mot</td>
<td>ROT</td>
<td>FIG</td>
<td>PIG</td>
<td>PUG</td>
<td>PUS</td>
<td>RED</td>
<td>BED</td>
<td>BID</td>
<td>BIN</td>
<td>RAD</td>
<td>RAT</td>
<td>VAT</td>
</tr>
<tr>
<td>GOD</td>
<td>ROD</td>
<td>ROT</td>
<td>RAT</td>
<td>DIG</td>
<td>BIG</td>
<td>BUG</td>
<td>BUS</td>
<td>WED</td>
<td>FED</td>
<td>FID</td>
<td>FIN</td>
<td>SOD</td>
<td>SAD</td>
<td>SAT</td>
</tr>
</tbody>
</table>

In these and the other WLSs and WLCs in this article, words which are a part of one or more ideal word ladders are shown in upper case letters, and words which are not a part of an ideal word ladder are shown in lower case letters.
Ideal and perfect proper WLCs for three-letter words are not difficult to construct, although care must be taken that all of the 12 WLSs in the WLC conform to the two-word sequence rule. Only a third of these WLSs need be shown to define the cube, however; for example, the following four WLSs, if placed atop one another in order from left to right, define an ideal proper WLC:

\[
\begin{align*}
\text{MAN MAD GAD GOD} & \quad \text{MAT mar mad mod} & \quad \text{BAT bar bad mad} & \quad \text{fat far fad dad} \\
\text{CAN CAD SAD SOD} & \quad \text{CAT car cad cod} & \quad \text{TAT tar tad cad} & \quad \text{gat gar gad lad} \\
\text{CAR cab cad COD} & \quad \text{CAP can cab COB} & \quad \text{TAP tan tab CAB} & \quad \text{gap gan gab LAB} \\
\text{CUR car cap cop} & \quad \text{CUP cap cat cot} & \quad \text{TUP tap tat cat} & \quad \text{TAP tan tab cab}
\end{align*}
\]

It is probably impossible to form a sublime proper WLS of three-letter words (or of words of any other length). Although the next WLC is shown in all upper case letters, it is not a sublime WLC, as only the four of its 12 constituent WLSs that are displayed are perfect; it is thus merely a perfect proper WLC.

\[
\begin{align*}
\text{FAN PAN PIN PIT} & \quad \text{GAN BAN BIN BIT} & \quad \text{CAN WAN WIN WIT} & \quad \text{BAN TAN TIN TIT} \\
\text{FIN PIN PUN PUT} & \quad \text{GIN BIN BUN BUT} & \quad \text{GAN BAN BIN BIT} & \quad \text{RAN PAN PIN PIT} \\
\text{FIG PIG PUG PUN} & \quad \text{GIG BIG BUG BUN} & \quad \text{GAG BAG BIG BIN} & \quad \text{RAG FAG FIG PIN} \\
\text{BIG RIG RUG RUN} & \quad \text{WIG DIG DUG DUN} & \quad \text{GIG BIG BUG BUN} & \quad \text{RIG FIG FUG FUN}
\end{align*}
\]

**Four-Letter Words.** Simple proper WLSs of three- or four-letter words are generally very easy to create, but may be less so if the object is to incorporate interesting or related words at the corner (and perhaps also the center) positions. An example of such a corner- and center-oriented simple WLS is shown below; the two squares in the next row are perfect proper WLSs.

\[
\begin{align*}
\text{love} & \quad \text{hove} & \quad \text{have} & \quad \text{hake} & \quad \text{hate} \\
\text{lave} & \quad \text{have} & \quad \text{hive} & \quad \text{hike} & \quad \text{hake} \\
\text{wave} & \quad \text{lave} & \quad \text{live} & \quad \text{like} & \quad \text{lake} \\
\text{gave} & \quad \text{cave} & \quad \text{lave} & \quad \text{lake} & \quad \text{wake}
\end{align*}
\]

\[
\begin{align*}
\text{WORD LORD LORE LONE LANE} & \quad \text{BANK BARK BARD BORD CORD} \\
\text{CORD BORD BORE BONE BANE} & \quad \text{BASK BALK BALD BOLD COLD} \\
\text{CARD BARD BARE BANE BINE} & \quad \text{BASE BALE BALL BOLL COLL} \\
\text{CARE BARE BARD BAND BIND} & \quad \text{BISE BILE BILL BELL CELL} \\
\text{CANE BANE BAND BARD BIRD} & \quad \text{WISE WILE WILL WELL TELL}
\end{align*}
\]

Ideal and perfect proper WLCs of four-letter words are somewhat tricky to devise, mainly due to the need to satisfy the two-word sequence rule in the 15 constituent WLSs of such cubes. At the top of the next page are the exploded views of two proper WLCs of four-letter words: the one in the left-hand column is ideal, and the one in the right-hand column is perfect.
Five- and Six-Letter Words. Here are two ideal proper WLSs of five-letter words and one ideal proper WLS of six-letter words:

HONOR HONER LONER LINER LIVER LIVED NAVAL NAVEl RAVEL RATEl RATES RITeS
HONER hosier loser loner lover loved NAVAL naves rates rates bates bites
HOVER hooper loper lover lower loved RAVEl raves paves pates mates mites
DOVER doper coper cover cower cowed RAVED raver paver pater mater miter
DIVER dover cover corer coder coded ROVED rover raver rater cater mater
DIVES doves coves cores codes codex ROBED roved raved rated rater water

VENdOR VENDER VENTER VESTER VESTED TESTED TASTED
vender render renter rester rested bested basted
mender tender tenter tester tested jested bested
mended tended tented tested tester jester bester
tended rended rented rested rester vester fester
tented rented vented vested vester westor pester
tested rested vested nested nester fester jester

As may be seen from these examples, the simplest (and perhaps only) way to make ideal proper WLSs of five- and six-letter words is to use mostly four-letter words with one- and two-letter suffixes added. It may
not be possible to devise a perfect proper WLS for five-letter words, and it is very probably impossible to devise one for six-letter or longer words. Likewise, an ideal or perfect proper WLC of five-letter words may be impossible to construct, and one of six-letter or longer words is very likely so.

**Patterned Simple Proper Word Ladder Squares and Cubes** In contrast, simple proper WLSs and WLCs of words seven or more letters long can readily be made. An easy way to do this is to take groups of words which differ from each other at only one or two letter positions and arrange them in accordance with certain general matrix patterns which comply with the two-word sequence rule. The pattern at top left below, for example, generates a simple proper WLC from any four words of the same length which differ from each other at only one letter position; the pattern at top right does the same from any six words of this kind. Any one of the individual matrices in these WLC patterns may also be a pattern for a simple proper WLS, as exemplified by the 36-word WLS at bottom.

```
1234 2413 3142 4321 1234 2413 3142 4321 1234 2413 3142 4321
2413 4321 1234 3142 1234 4321 1234 3142 2413 4321 1234 3142
3142 1234 4321 2413 3142 1234 4321 2413 3142 1234 4321 2413
4321 3142 2413 1234 4321 3142 2413 1234 4321 3142 2413 1234
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dustinesses fustinesses gustinesses lustinesses mustinesses rustinesses
fustinesses lustinesses rustinesses dustinesses lustinesses mustinesses
lustinesses rustinesses fustinesses mustinesses dustinesses lustinesses
lustinesses dustinesses mustinesses fustinesses rustinesses gustinesses
mustinesses gustinesses dustinesses rustinesses lustinesses mustinesses
lustinesses lustinesses gustinesses fustinesses rustinesses dustinesses

Words of an even wider wingspan (all from Webster's Third) flock together in this patterned simple proper WLS of sixteen 15-letter words:

```
lightlessnesses nightlessnesses rightlessnesses sightlessnesses
  nightlessnesses sightlessnesses lightlessnesses rightlessnesses
  rightlessnesses lightlessnesses sightlessnesses nightlessnesses
  sightlessnesses rightlessnesses nightlessnesses lightlessnesses
```

Matrix patterns for generating simple proper WLSs and WLCs of other sizes exist, as do similar patterns for groups of words which differ at two letter positions. These rather monotonous arrays are of interest chiefly because they permit relatively long words to be used in proper WLSs and WLCs.

**Heterogeneous Word Ladder Squares and Cubes** Heterogeneous word ladder arrays are a variety of proper word ladder array in which no two words may be the same. Owing to their nonredundant character they are perhaps the most esthetically pleasing of the word ladder arrays,
but for the same reason they are also relatively difficult to devise and somewhat restricted in scope. A number of the ideal and perfect WLSs of three- and four-letter words shown previously have been heterogeneous, but it seems doubtful that any ideal or perfect heterogeneous WLS of five-letter or longer words could be formed. Entirely unexplored is the question of how large the largest possible simple heterogeneous WLSs of various word lengths can be. Larger simple heterogeneous WLSs of four-letter words than this 100-word specimen, for instance, can doubtless be assembled—but just how much larger?

wire hire dire dice pice pile wile wine sine site ware hare dare dace pace pale wale wane same sate fare pare mare mace race rale gale gane dane date fane pane mane mate rate rave gave game dame dale vane hane bane bate late lave cave came tame tale vale hale bale bade lade lane cane cake take tare vole hole bole bode lode lone cone coke toke tore tole mole rode node none pone poke moke more tile mile rile ride ride nine pine pike mire vile sile bile tide tine bine bike like lile

An eleventh column, cite-cate-fate-fale-cale-care-core-sore-sire-fire, could be added to this square on the right, but then it would no longer be conceptually square, having become a rectangle in plan as well as appearance.

Heterogeneous WLCs are feasible, but are considerably more difficult to devise than nonheterogeneous WLCs. Only at the cost of including a disagreeably-large number of exotic words, for example, was I able to complete the following simple heterogeneous WLCs of three-letter and four-letter words:

cat nat rat wat bat fat gat tat bar far gar tar car mar par war
cag nag rag wag bag fag gag tag ban fan gan tan can man pan wan
cay nay ray way bay fay gay tay bad fad gad tad cad mad pad wad
jay kay may pay day hay lay say dad had lad sad rad vad yad zad

bade cade fade made bake cake fake make bane cane fane mane
bale cale fale male hane rane sane wane
hake rake sake wake have rave save wave
bare care fare mare bars cars fars mars
bate cate fate mate bats cats fats mats
hate rate sate wate hats rats sats wats
hage rage sage wage hags rags sags wags

It is probably possible to form simple heterogeneous WLCs of four-letter words in which all of the words can accept identical one- to four-letter suffixes; if it is, simple heterogeneous WLCs of words up to eight letters in length can obviously then be made from it. It is also possible
(barely) that ideal or perfect heterogeneous WLCs of three- or four-letter words can be fashioned.

IMPROPER WORD LADDER SQUARES AND CUBES

Improper word ladder arrays contain repeated segments of the same word ladder in their rows and/or columns, and may in fact be entirely composed of such repetitive segments. Given the word ladder PASS-PANS-PANT-PINT-DINT-DIRT-DIRK-DARK-HARK-HANK-HAND-HIND-FIND one can simply write down progressively offset segments of the ladder to obtain the perfect WLS at left below. A similar procedure generates from this WLS the WLC shown at right below. To save space, the cube is depicted symbolically, with the letters A through M representing, in order, the words in the PASS/FIND ladder.

<table>
<thead>
<tr>
<th>PASS</th>
<th>PANS</th>
<th>PANT</th>
<th>PINT</th>
<th>DINT</th>
<th>DIRT</th>
<th>DIRK</th>
<th>DARK</th>
<th>HARK</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABCDE</td>
<td>BCDEF</td>
<td>CDEFG</td>
<td>DEFGH</td>
<td>EFGHI</td>
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</tr>
<tr>
<td>PENNSA</td>
<td>PANTES</td>
<td>PEINTS</td>
<td>DENINTS</td>
<td>DERNITNS</td>
<td>DERNPNTS</td>
<td>DERNPINTS</td>
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<td>PENNSA</td>
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<td>DERNITNS</td>
<td>DERNPNTS</td>
<td>DERNPINTS</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

It may be worth noting that the cube thus generated is an example of a sublime WLC, in which all 15 WLSs are perfect and all 75 word ladders are ideal. Not every word ladder will generate a perfect WLS and a sublime WLC in this manner; only those which incorporate cycles in which each letter position changes once per cycle, and the changes are in the same order in each cycle, will do so. In the PASS/FIND ladder, for instance, the sequence of letter position changes is 3421, 3421, 3421. It might also be mentioned that it is possible to generate improper WLSs and WLCs with fewer different words by utilizing circular word ladders. As an example, the circular ladder BANE-BONE-CONE-CORE-CORD-CARD-BARD-BAND-BANE will generate a perfect WLS and a sublime WLC with only eight different words, as compared to the PASS/FIND ladder's total of thirteen.

Logologically, there is an evident need to distinguish such automatic constructions as these from their non-trivial counterparts, the proper word ladder squares and cubes; the question is what exactly that distinction should be. An esthetically attractive alternative to the two-word sequence rule might be to require that proper word ladder arrays be heterogeneous. But such a constraint would, I believe, be severely and unnecessarily limiting. On the other hand, the construction of proper WLSs and WLCs could be made much easier if some more lenient rule which, say, prohibited only three-word sequences from appearing more than once in the rows or columns of a proper WLS, or one which allowed a two-word sequence to appear twice but not three times in the rows or columns of a proper square, were to be adopted. Alas, such arbitrary rules as these have a decidedly unesthetic feeling to them. To me, the rule which permits any two-word sequence to occur only once in the rows and only once in the columns of a proper WLS combines a reasonable esthetic appeal with an acceptable degree of difficulty. But it
is not the only rule available, nor is it necessarily the one best suited to every taste.

NONSTANDARD WORD LADDER SQUARES AND CUBES

As previously noted, nonstandard word ladder arrays are those in which a word may appear more than once in the same ladder. The greater latitude afforded by such arrays may serve a variety of creative purposes, such as facilitating the arrangement of key words in a WLS (first example below), enabling the construction of WLSs in which all of the ladders are circular (second example below), and permitting the fabrication of WLSs in which all of the ladders are palindromic:

checker chickner chucker clucker plucker
chickner whicker Chicner chucker clucker
chucker Chickner chicken chick cricker
chucker chickner chucker clicker clocker
shucker shocker chickcr clocker crocker

corner conner canner conner corner
conner canner canter canner conner
conner canter center canter canner
conner canter canter canner conner
conner conner canner conner corner

keel keep peep peek leek leper lever rever revel repel
keep neep peep peen peek lever rever refer rever revel
peep peep peep peep peep rever refer rever refer rever
peek peen peep neep keep revel rever refer rever lever
leek peep peep keep keel repel revel rever lever leper

Nonstandard WLCs having the same properties may also be devised. An example of a fully palindromic WLC, in which all of the 35 ladders are palindromic, is displayed below. Since the cube is fully palindromic, it is adequately defined by showing just the first three squares in a stack; the fourth and fifth squares in the stack are obtained by simply reversing the individual words in the second and first squares, respectively.

got not tot ton togt gut nut tut tun tug tut tut tot tut tut
pot tot tut tot top put tut tat tut tut tut tat tit tot tat
tot tut tat tut tot tut tat tot tat tut tut tat tit tut tit tat
top tot tut tot pot tup tut tat tut put tut tat tit tat tut
tog ton tot not got tug tun tut nut gut tut tut tot tut tut

PARAMETERS UNKNOWN

As broad as this overview of word ladder squares and cubes has been, it is still far from being a complete survey of the entire field of word ladder arrays. Omitted from the discussion have been such topics of potential logological interest as arrays which are not square or
cubical in shape, arrays which are not orthogonal in structure, word ladder hypercubes, arrays in which word ladders are not confined to a single row or column, and arrays in which the letters in the words have been assigned numerical values.

With respect to the topic with which this article has been primarily concerned, the standard proper word ladder squares and cubes which are governed by the two-word sequence rule, there will no doubt always be a number of more or less open questions relating to maximal dimensions. At the moment, these include the following:

1. Can a regulation-sized simple proper WLS of words longer than 15 letters be formed?
2. Can an ideal proper WLS of words longer than six letters be formed?
3. Can a perfect proper WLS of words longer than four letters be formed?
4. What are the largest simple proper WLSs of different word lengths which can be formed?
5. Can a regulation-sized simple proper WLC of words longer than 15 letters be formed?
6. Can an ideal proper WLC of words longer than four letters be formed?
7. Can a perfect proper WLC of words longer than four letters be formed?
8. Can a sublime proper WLC of any word length be formed?
9. What are the largest simple heterogeneous WLSs of different word lengths which can be formed?
10. Can any ideal or perfect heterogeneous WLSs of words of five or more letters be formed?
11. What are the largest simple heterogeneous WLCs of different word lengths which can be formed?
12. Can any ideal, perfect or sublime heterogeneous WLC be formed?

Clearly, much of this terrain remains terra incognita.

REFERENCES

All of the words used in the WLSs and WLCs in this article are dictionary-listed, and may be found in one or more of the following: Webster's Third New International Dictionary, the Oxford English Dictionary, Webster's New International Dictionary Second Edition ('chocker'), and the New Dictionary of American Slang ('hoser').